

REVIEW PROBLEMS

PHYSICS 121

Chapters 1-5

1. Two vectors, $\vec{A}$ and $\vec{B}$, have the components

$$A_x = 3 \quad A_y = 4 \quad A_z = 0 \quad ; \quad B_x = -2 \quad B_y = 5 \quad B_z = 0.$$

Define a new vector: $\vec{C} = \vec{A} - \vec{B}$.

- (a) Find the magnitude and direction of $\vec{C}$.
- (b) Compute $\vec{A} \cdot \vec{C}$ and $\vec{A} \times \vec{C}$.
- (c) Find the angle between A and C .

2. A jet airplane is flying at a constant speed of $150 \frac{m}{s}$, at an unknown angle above the horizontal. Its shadow is measured to be moving along the ground at $70 \frac{m}{s}$. (In other words, the x-component of its velocity is $70 \frac{m}{s}$.)

- (a) What is the angle θ of its velocity above the horizon?
- (b) What is the change in altitude of the jet during a 2 second interval?
- (c) What is the total distance the jet travels during these 2 seconds?

3. Now suppose the jet in the previous problem begins to accelerate at a rate of $20 \frac{m}{s^2}$. The acceleration and velocity are both in the same direction as the velocity in problem 2, and the initial speed is $150 \frac{m}{s}$.

- (a) Find the change in altitude during the first 2 seconds.
- (b) How long does it take the jet to travel a distance of 10,000 meters?
- (c) What is the value of v_y after the jet has traveled a **vertical** distance of 2 miles?

4. A batter hits a baseball at a 30° angle above the horizontal, with an initial speed of $25 \frac{m}{s}$. Take the point where the ball hits the bat to be the origin of the coordinate system.

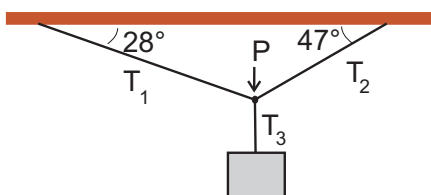
- (a) If an outfielder tries to catch the ball, how far from the batter should he be standing? (Assume the height of the ball when caught is the same as the height at which it left the bat.)
- (b) How long does the ball take to reach the outfielder?
- (c) What is the maximum height the ball reaches?
- (d) What is the speed when the ball is at a height of 2 meters above the height of the bat?

5. A skier is starting down a slope which is at 20° above the horizontal. She starts with an initial speed of $1 \frac{m}{s}$. Ignore friction and assume that $m = 50$ kg.

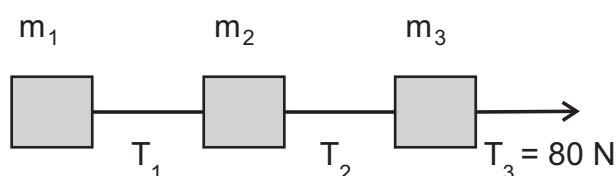
(a) Find the magnitudes of her acceleration and of the normal force.

(b) What is her total speed after 2 seconds?

6. A 15 kg mass is supported by a set of ropes as shown below. By looking at the forces exerted at point P, find the tension in each of the three ropes.



7. Three masses are connected as shown below. The masses are $m_1 = 12$ kg, $m_2 = 24$ kg, $m_3 = 36$ kg. If a force $T_3 = 80$ N is exerted toward the right on m_3 , find the acceleration of the blocks and the tensions T_1 and T_2 in the two connecting ropes. (Ignore any friction.)



8. A train is moving at a speed of 10 miles per hour. A child on the train tosses a baseball into the air at a speed of $4 \frac{m}{s}$, at 80° above the horizon, as seen from the train. How fast and at what angle does the ball seem to be moving, as seen by a stationary observer looking in the window of the train from the station platform? (Hint: this is really just a vector addition problem.)

9. A particle of mass $m = 3 \text{ kg}$ is subjected to a time-dependent force $F(t)$. As a result, it moves along the x -axis with a position at time t given by the function $x(t) = \frac{3}{20}t^5 - \frac{1}{6}t^3 + 2t$. (Parts b and c require derivatives, which won't be needed on test.)

- (a) What is the displacement between the times $t = 0$ and $t = 5 \text{ s}$?
- (b) Find the velocity as a function of time. What is the speed v at $t = 5 \text{ s}$?
- (c) Find the force acting on the particle, as a function of time.

10. An electron (mass $m = 9.11 \times 10^{-31} \text{ kg}$) is initially at rest. An electric field exerts a force on the electron causing it to accelerate at a constant rate toward a neutron 1 mm away. The force is of magnitude $2 \times 10^{-28} \text{ N}$.

- (a) How fast is the electron moving just before it collides with the neutron?
- (b) How much time does the electron take to reach the neutron?
- (c) Suppose the same electron is now in a circular orbit around a proton, forming a hydrogen atom. The radius of the orbit is $5.3 \times 10^{-11} \text{ m}$. If the speed of the electron is $2.5 \times 10^6 \frac{\text{m}}{\text{s}}$ (a bit less than 1% of the speed of light), find the centripetal acceleration.
- (d) How many times per second does the electron in part (c) circle the proton?

11. A block mass $m = 5 \text{ kg}$ is placed on a surface with coefficients of friction $\mu_k = 0.4$ and $\mu_s = 0.6$.

- (a) If we start tilting the surface upward to form a ramp, what is the maximum angle it can be tilted above the horizontal before the block begins to slide?
- (b) Now we tilt the ramp to 40° and release the block from rest. What is its acceleration as it slides down the ramp?

12. A car is going over the top of a hill. The hilltop looks like part of a sphere, with a radius of curvature of $r = 40 \text{ m}$. What is the maximum speed that the car can have before it loses contact with the road as it goes over the top?

① a) $C_x = A_x - B_x = 3 - (-2) = 5$

$C_y = A_y - B_y = 4 - 5 = -1$

$C_z = A_z - B_z = 0$

$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(5)^2 + (-1)^2 + (0)^2} = \sqrt{26} = \boxed{5.1}$

$\theta = \tan^{-1} \frac{C_y}{C_x} = \tan^{-1} \left(\frac{-1}{5} \right) = \boxed{-11.31^\circ}$

b) $\vec{A} \cdot \vec{C} = A_x C_x + A_y C_y + A_z C_z = (3)(5) + (4)(-1) + (0)(0) = \boxed{11}$

$\vec{A} \times \vec{C} = (A_y C_z - A_z C_y) \hat{i} + (A_z C_x - A_x C_z) \hat{j} + (A_x C_y - A_y C_x) \hat{k}$
 $= [4(0) - (0)(-1)] \hat{i} + [(0)(3) - (3)(0)] \hat{j} + [(3)(-1) - (4)(5)] \hat{k}$
 $= \boxed{-23 \hat{k}} \text{ (ALONG NEGATIVE } z\text{-AXIS)}$

c) WE NOW FROM ABOVE THAT $C = 5.1$ AND $\vec{A} \cdot \vec{C} = 11$.
 ALSO, $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(3)^2 + (4)^2 + (0)^2} = 5$.

so! $\vec{A} \cdot \vec{C} = AC(\cos \theta) \Rightarrow \theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{C}}{AC} \right) = \cos^{-1} \left(\frac{11}{(5.1)(5)} \right) = \boxed{64.4^\circ}$

② a) $v = 150 \frac{m}{s}, v_x = 70 \frac{m}{s}$

$v_x = v \cos \theta \Rightarrow \theta = \cos^{-1} \frac{v_x}{v} = \cos^{-1} \frac{70}{150} = \boxed{62.18^\circ}$

b) $v_y = v \sin \theta = (150) \sin 62.18^\circ = 132.66 \frac{m}{s}$

$\Delta y = v_y \Delta t = (132.66 \frac{m}{s})(2s) = \boxed{265.32 m}$

c) DISTANCE = (TOTAL SPEED) $\Delta t = (150 \frac{m}{s})(2s) = \boxed{300 m}$

↑ CAN DO THIS ONLY BECAUSE THERE IS NO ACCELERATION

$$(3) \quad v_0 = 150 \frac{m}{s}, \quad \theta = 62.18^\circ, \quad a = 20 \frac{m}{s^2}$$

$$a_y = a \sin \theta = 20 \sin 62.18^\circ = 17.69 \frac{m}{s^2}$$

FROM THE LAST PROBLEM, $v_{0x} = 70 \frac{m}{s}$ & $v_{0y} = 132.66 \frac{m}{s}$.

$$a) \quad y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 = (132.66)(2) + \frac{1}{2} (17.69)(2)^2 = \boxed{300.7 \text{ m}}$$

b) DISTANCE TRAVELED:

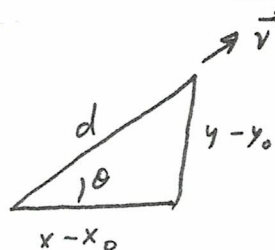
$$d = v_0 t + \frac{1}{2} a t^2 \Rightarrow \frac{1}{2} a t^2 + v_0 t - d = 0$$

SOLVE QUADRATIC EQUATION:

$$t = \frac{-v_0 \pm \sqrt{v_0^2 + 2ad}}{a}$$

THE TIME MUST BE POSITIVE, SO KEEP ONLY THE SOLUTION WITH THE + SIGN:

$$t = \frac{-150 \pm \sqrt{(150)^2 + 2(20)(10000)}}{20} = \boxed{25 \text{ s}}$$



$$c) \quad y - y_0 = (2 \text{ miles}) \left(\frac{1610 \text{ m}}{1 \text{ mile}} \right) = 3220 \text{ m}$$

$$v_x^2 = v_{0x}^2 + 2a_y(y - y_0) \rightarrow v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)}$$

so:

$$v_y = \sqrt{(132.66)^2 + 2(17.69)(3220)} = \boxed{362.65}$$

$$\textcircled{4} \quad \theta = 30^\circ, \quad v_0 = 25 \frac{\text{m}}{\text{s}} \quad \Rightarrow \quad \begin{cases} v_{0x} = v_0 \cos \theta = 21.65 \frac{\text{m}}{\text{s}} \\ v_{0y} = v_0 \sin \theta = 12.50 \frac{\text{m}}{\text{s}} \end{cases}$$

$$\text{a) } R = \frac{v_0^2 \sin 2\theta}{g} = \frac{(25)^2 \sin 60^\circ}{9.8} = \boxed{55.23 \text{ m}}$$

$$\text{b) } x = x_0 + v_{0x} t \quad \xrightarrow{\text{SOLVE FOR } t} \quad t = \frac{x - x_0}{v_{0x}}$$

$$\text{BUT } x - x_0 = R, \text{ so } t = \frac{R}{v_{0x}} = \frac{55.23 \text{ m}}{21.65 \frac{\text{m}}{\text{s}}} = \boxed{2.55 \text{ s}}$$

$$\text{c) USE } v_y^2 = v_{0y}^2 - 2g(y - y_0). \text{ AT HIGHEST POINT } v_y = 0, \text{ so:}$$

$$0 = v_{0y}^2 - 2g(y - y_0).$$

$$\text{SOLVE FOR HEIGHT: } y - y_0 = \frac{v_{0y}^2}{2g} = \frac{(12.5)^2}{2(9.8)} = \boxed{7.97 \text{ m}}$$

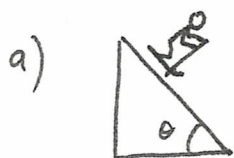
$$\text{d) NOW, } y - y_0 = 2 \text{ m.}$$

$$v_x^2 = v_{0x}^2 = (21.65)^2 = 468.72 \frac{\text{m}^2}{\text{s}^2}$$

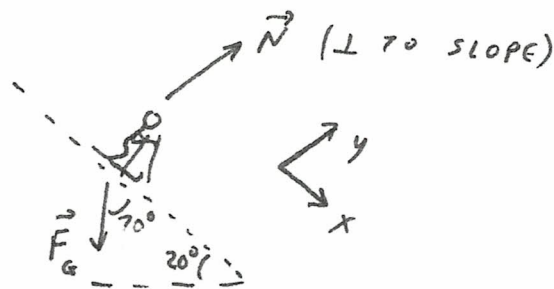
$$v_y^2 = v_{0y}^2 - 2g(y - y_0) = (12.5)^2 - 2(9.8)(2) = 117.05 \frac{\text{m}^2}{\text{s}^2}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{468.72 + 117.05} = \boxed{24.2 \text{ m/s}}$$

⑤ $\theta = 20^\circ$, $v_0 = 1 \frac{m}{s}$, $m = 50 \text{ kg}$



FORCE DIAGRAM!



TAKE X-AXIS PARALLEL TO SLOPE, Y-AXIS $\perp$ TO IT.

COMPONENTS: $N_x = 0$

$F_{gx} = mg \cos 70^\circ$

$a_x = a$

$N_y = N$

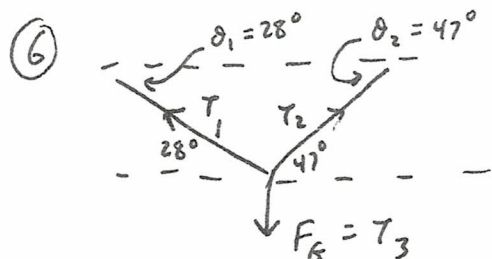
$F_{gy} = -mg \sin 70^\circ$

$a_y = 0$

$\Sigma F_x = ma_x \rightarrow mg \cos 70^\circ = ma \rightarrow a = g \cos 70^\circ = \boxed{3.35 \frac{m}{s^2}}$

$\Sigma F_y = ma_y \rightarrow N - mg \sin 70^\circ = 0 \rightarrow N = mg \sin 70^\circ = \boxed{460.45 \text{ N}}$

b) $v = v_0 + at = 1 \frac{m}{s} + (3.35 \frac{m}{s^2})(2s) = \boxed{7.7 \frac{m}{s}}$



$T_3 = mg = (15 \text{ kg})(9.8 \frac{m}{s^2}) = \boxed{147 \text{ N}}$

$a_x = a_y = 0$ $m = 15 \text{ kg}$

$\Sigma F_x = ma_x \rightarrow T_2 \cos 47^\circ - T_1 \cos 28^\circ = 0$

SO: $T_2 = T_1 \left(\frac{\cos 28^\circ}{\cos 47^\circ} \right)$ (1)

$\Sigma F_y = ma_y \rightarrow T_1 \sin 28^\circ + T_2 \sin 47^\circ - mg = 0$ (2)

SUBSTITUTE (2) INTO (1):

$T_1 \sin 28^\circ + T_1 \left(\frac{\cos 28^\circ}{\cos 47^\circ} \right) \sin 47^\circ - mg = 0$

USE $\tan = \frac{\sin}{\cos}$, & REARRANGE:

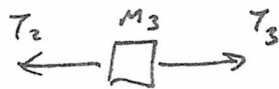
$T_1 = \frac{mg}{\sin 28^\circ + (\cos 28^\circ) \tan 47^\circ} = \boxed{103.82 \text{ N}}$

$T_2 = T_1 \left(\frac{\cos 28^\circ}{\cos 47^\circ} \right) = \boxed{134.4 \text{ N}}$

- ⑦ (i) TOTAL MASS: $M = M_1 + M_2 + M_3 = 12 + 24 + 36 = 72 \text{ kg}$
 THIS TOTAL MASS IS ACCELERATED BY EXTERNAL FORCE
 OF $T_3 = 80 \text{ N}$. SO, BY NEWTON'S 2nd LAW:

$$T_3 = Ma \Rightarrow a = \frac{T_3}{M} = \frac{80 \text{ N}}{72 \text{ kg}} = \boxed{1.11 \frac{\text{m}}{\text{s}^2}}$$

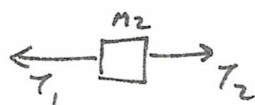
- (ii) NOW LOOK AT M_3 :



$$\Sigma F = m_3 a \rightarrow T_3 - T_2 = m_3 a$$

$$T_2 = T_3 - m_3 a = 80 - (36)(1.11) = \boxed{40.04 \text{ N}}$$

- (iii) LOOK AT M_2 :



$$\Sigma F = m_2 a \Rightarrow T_2 - T_1 = m_2 a$$

$$T_1 = T_2 - m_2 a = 40.04 - (24)(1.11) = \boxed{13.4 \text{ N}}$$

- ⑧ TRAIN RELATIVE TO STATION: $\begin{cases} v_{TSX} = (10 \frac{\text{MILES}}{\text{HR}}) (\frac{1 \text{ HR}}{3600 \text{ S}}) (\frac{1610 \text{ M}}{1 \text{ MILE}}) = 4.47 \frac{\text{m}}{\text{s}} \\ v_{TSY} = 0 \end{cases}$

BALL RELATIVE TO TRAIN:

$$(v_{BT} = 4 \frac{\text{m}}{\text{s}}, \theta = 80^\circ)$$

$$\begin{cases} v_{BTX} = v_{BT} \cos \theta = 4 \cos 80^\circ = .695 \frac{\text{m}}{\text{s}} \\ v_{BTY} = v_{BT} \sin \theta = 4 \sin 80^\circ = 3.94 \frac{\text{m}}{\text{s}} \end{cases}$$

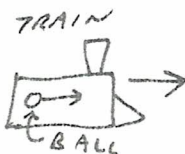
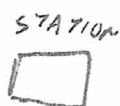
BALL RELATIVE TO STATION:

$$v_{BSX} = v_{BTX} + v_{TSX} = .695 + 4.47 = 5.17 \frac{\text{m}}{\text{s}}$$

$$v_{BSY} = v_{BTY} + v_{TSY} = 3.94 + 0 = 3.94 \frac{\text{m}}{\text{s}}$$

$$v_{BS} = \sqrt{v_{BSX}^2 + v_{BSY}^2} = \sqrt{(5.17)^2 + (3.94)^2} = \boxed{6.5 \frac{\text{m}}{\text{s}}}$$

$$\theta = \tan^{-1} \left(\frac{v_{BSY}}{v_{BSX}} \right) = \tan^{-1} \left(\frac{3.94}{5.17} \right) = \boxed{37.31^\circ}$$



$$\begin{array}{ccc} \xrightarrow{v_{TS}} & \xrightarrow{v_{BT}} & \\ \hline \xrightarrow{v_{BS} = v_{TS} + v_{BT}} & & \end{array}$$

⑨ $m = 3 \text{ kg}$, $x(t) = \frac{3}{20} t^5 - \frac{1}{6} t^3 + 2t$.

a) $\Delta x = x(t) - x(t_0)$
 $= \left[\underbrace{\frac{3}{20} (5)^5 - \frac{1}{6} (5)^3 + 2(5)}_{x \text{ at } t=5s} \right] - \underbrace{[0]}_{x \text{ at } t_0=0}$
 $= \boxed{457.9 \text{ m}}$

b) $v(t) = \frac{dx}{dt} = \frac{d}{dt} \left(\frac{3}{20} t^5 - \frac{1}{6} t^3 + 2t \right) = \boxed{\frac{3}{4} t^4 - \frac{1}{2} t^2 + 2}$

At $t = 5s$: $v(5) = \boxed{458.25 \frac{\text{m}}{\text{s}}}$

c) ACCELERATION: $a(t) = \frac{dv}{dt} = 3t^3 - t$.

FORCE: $F(t) = m a(t) = \boxed{9t^3 - 3t}$

⑩ $m = 9.11 \times 10^{-31} \text{ kg}$, $v_0 = 0$, $x - x_0 = \underbrace{1 \text{ mm}}_{=.001 \text{ m}}$, $F = 2 \times 10^{-28} \text{ N}$

a) $a = \frac{F}{m} = \frac{2 \times 10^{-28}}{9.11 \times 10^{-31}} = 219.5 \frac{\text{m}}{\text{s}^2}$

$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{0 + 2(219.5)(.001)} = \boxed{.66 \frac{\text{m}}{\text{s}}}$

b) $v = v_0 + at \rightarrow t = \frac{v - v_0}{a} = \frac{.66 \frac{\text{m}}{\text{s}}}{219.5 \frac{\text{m}}{\text{s}^2}} = \boxed{.003s}$

c) NOW: $r = 5.3 \times 10^{-11} \text{ m}$, $v = 2.5 \times 10^6 \frac{\text{m}}{\text{s}}$ (constant)

$a_c = \frac{v^2}{r} = \frac{(2.5 \times 10^6)^2}{5.3 \times 10^{-11}} = \boxed{1.2 \times 10^{23} \frac{\text{m}}{\text{s}^2}}$

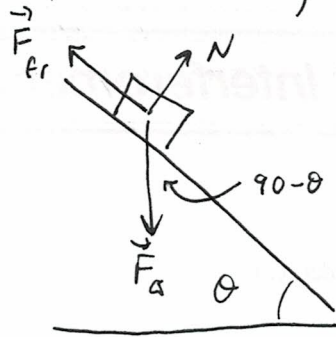
d) TIME FOR ONE TRIP AROUND (PERIOD, T):

$\underbrace{(2\pi r)}_{\text{DISTANCE}} = vT \rightarrow T = \frac{2\pi r}{v} = \frac{2\pi(5.3 \times 10^{-11})}{2.5 \times 10^6} = 1.3 \times 10^{-16} \text{ s}$

OF REVOLUTIONS PER SECOND: $\frac{1}{T} = \boxed{7.5 \times 10^{15} \frac{\text{REVS}}{\text{s}}}$

11

a) $m = 5 \text{ kg}$, $\mu_k = 0.4$, $\mu_s = 0.6$



TAKE AXES $\parallel$ & $\perp$ TO RAMP;



AT REST: $\vec{a} = 0$

$$\Sigma F_y = ma_y \rightarrow N - mg \sin(90 - \theta) = 0 ,$$

so: $N = mg \sin(90 - \theta)$, or $\boxed{N = mg \cos \theta}$ (A)

$$\Sigma F_x = ma_x \rightarrow mg \cos(90 - \theta) - F_{fr} = ma ,$$

or: $\boxed{mg \sin \theta - \mu_s N = ma}$ (B)

PLUG (A) INTO (B):

$$\cancel{mg} \sin \theta - \mu_s \cancel{mg} \cos \theta = \cancel{m}a \rightarrow \boxed{g \sin \theta - \mu_s g \cos \theta = a}$$
 (C)

For $a = 0$:

$$g \sin \theta - \mu_s g \cos \theta = 0 \rightarrow \mu_s = \tan \theta \rightarrow \theta = \tan^{-1} \mu_s = \boxed{30.96^\circ}$$

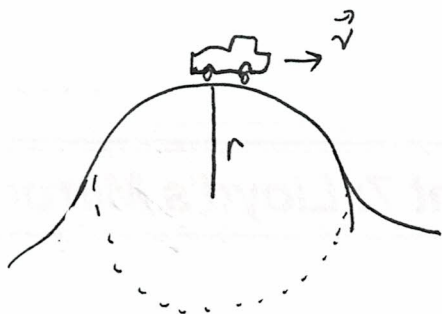
b) DO SAME AS PART a, BUT REPLACE $\mu_s \rightarrow \mu_k$.
EQUATION (C) THEN BECOMES:

$$g (\sin \theta - \mu_k \cos \theta) = a$$

so:

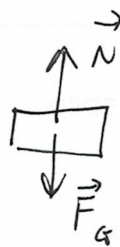
$$a = (9.8) (\sin 40^\circ - \mu_k \cos 40^\circ) = \boxed{3.3 \frac{m}{s^2}}$$

(12)



$$r = 40 \text{ m}$$

AT TOP OF HILL:



$$\sum F_y = ma_y \rightarrow N - mg = -m \frac{v^2}{r}$$

IF THE CAR LOSES CONTACT WITH THE ROAD, THEN THE NORMAL FORCE DROPS TO ZERO: $N = 0$.

$$\text{so: } mg = m \frac{v^2}{r} \rightarrow v = \sqrt{gr} = \sqrt{(9.8)(40)}$$

$$= \boxed{19.8 \frac{\text{m}}{\text{s}}}$$

$$\left(= 44.3 \frac{\text{MILES}}{\text{HR}} \right)$$

Physics 121
REVIEW PROBLEMS
Chapters 6-11

[1] The ball in a pinball machine is fired by allowing it to come to rest against a compressed spring, and then releasing the spring.

(a) If a spring of spring constant $k = 1.5 \frac{N}{m}$ is compressed by .08 m, what will be the speed it gives to a pinball of mass .01 kg?

(b) What force is required to hold the spring in the compressed position?

[2] (a) A rock of mass .2 kg is shot into the air by a slingshot at an unknown angle, with initial speed $v_0 = 9 \frac{m}{s}$. What is the rock's speed when it is 2 m above the height from which it was fired?

(b) The rock is retrieved, and fired again. This time it is shot straight upward (no horizontal component), with an unknown initial speed. If the slingshot acts like a spring with an effective spring constant $k = 10 \frac{N}{m}$, how much must it be stretched in order to shoot the rock to a height of 4 m?

[3] Suppose a shuttle craft from the starship Enterprise is powered by firing a stream of ions out the back. The mass of the craft is $m_s = 2 \times 10^5$ kg. The rate at which the ions are expelled is $\frac{\Delta m}{\Delta t} = 10 \frac{kg}{s}$, and their speed is $v_{ION} = 3 \times 10^6 \frac{m}{s}$.

(a) What is the force exerted on the shuttle by the ion stream?

(b) If the shuttle started at rest, what is its speed and momentum after 2 seconds?

(c) Suppose that a while later the shuttle is cruising at a constant speed of $v_{si} = 10^4 \frac{m}{s}$. At this point, it collides with a ship of the Zrrgf Empire, which was initially stationary and has a mass $m_z = 4 \times 10^4$ kg. If the collision is completely inelastic, what is the final speed of the two ships?

(d) Suppose instead that the Zrrgf captain saw the shuttle coming in time to switch on the shields. Now the collision is elastic, and the two ships bounce off of each other. What is the final speed of each? (Assume that all the velocities are along the same line, so that the problem is one-dimensional.)

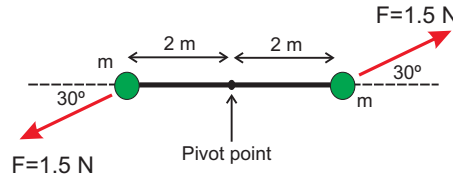
[4] A hammer strikes a nail of mass 5 grams (.005 kg). The hammer and nail are in contact for a time of .002 seconds, and afterwards the nail has a final speed of $15 \frac{m}{s}$. If the

nail was initially at rest, find the average force exerted by the hammer on the nail.

[5] Two small objects, each of mass $m=2$ kg, are connected by a rod of length 4 meters. The mass of the rod is negligible, and it is pivoted at the center, so it can rotate.

(a) Find the moment of inertia about the center of the rod.

(b) Now suppose two forces, both of size 1.5 N, are applied to the masses at an angle of 30° from the rod, as shown. Find the angular acceleration α of the masses.



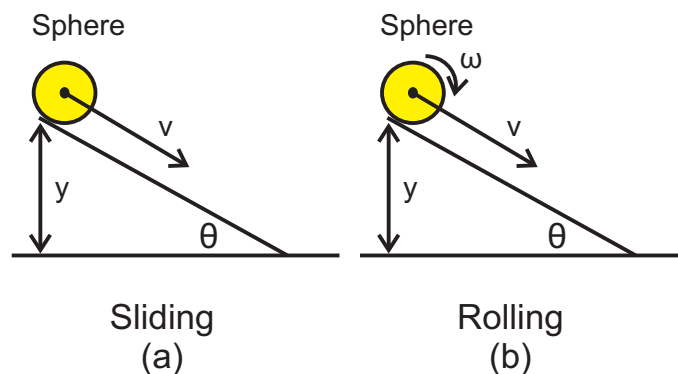
(c) Find the linear and angular speeds (v and ω) of the masses after 3 seconds. (Assume they start from rest.)

[6] A solid cylinder is pivoted to rotate about its center. The cylinder has mass $m=12$ kg and radius $r=.8$ m. It is initially spinning at an angular speed of $\omega_0 = 10 \frac{rad}{s}$.

(a) Find the torque τ required to bring the cylinder to rest in 20 seconds.

(b) How much work was done in bringing the cylinder to rest?

[7] A sphere of mass $M = 3$ kg and radius $R = .2$ m is at the top of a ramp of height $y = 2$ m. Find the speed of the ball at the bottom of the ramp, assuming (a) it slides without friction, and (b) it rolls without slipping. Why is the answer to (a) larger?



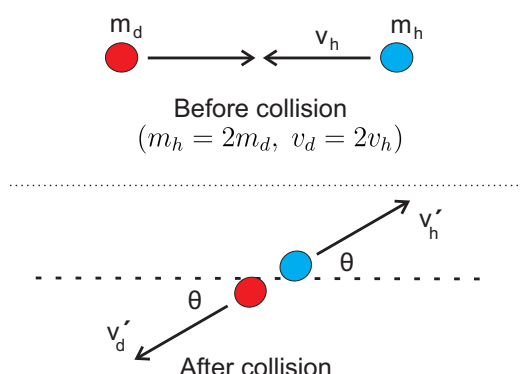
[8] A skier starts from rest at the top of a hill inclined at 20° from the horizontal. The coefficient of kinetic friction is $\mu_k = .12$ and the distance down the hill is 2000 m . (Note that this is the *length* of the slope, *not* the height.)

(a) What is the minimum coefficient of *static* friction that would prevent the skier from sliding down the hill? (Or, in other words, what is the maximum coefficient that would still allow skiing to take place?)

(b) Now suppose μ_s is less than the value found in part (a), so that the skier can actually ski down the hill. Assume that the person's mass is 60 kg . How much work is done (i) by gravity and (ii) by friction as the skier descends to the bottom?

(c) Find the kinetic energy the skier will have at the bottom of the hill.

[9] Inside a nuclear reactor, a deuterium nucleus collides head-on with a helium nucleus. (Deuterium is a heavy isotope of hydrogen, with an extra neutron.) The two nuclei initially have speeds $v_d = 2 \times 10^8 \frac{\text{m}}{\text{s}}$ and $v_h = 10^8 \frac{\text{m}}{\text{s}}$, so that $v_d = 2v_h$. Helium has twice the mass of deuterium, $m_h = 2m_d$. The nuclei bounce elastically off each other, and after the collision they both move off at angle θ from their initial line of approach, as shown. Find the speeds v'_d, v'_h at which they move apart after the collision. Can θ be found as well?



[10] A squid of mass 2.2 kg propels itself forward by expelling a stream of water from its bladder. Suppose that the water is moving at constant speed $v_w = 3.0 \frac{\text{m}}{\text{s}}$ and is expelled at a rate of $.1 \frac{\text{kg}}{\text{s}}$.

(a) How large is the force exerted on the squid by the expelled water?

(b) If the mass of the squid is *decreasing* at the same rate ($.1 \frac{\text{kg}}{\text{s}}$) during this period, how fast is the squid accelerating at a moment when its speed is $v_s = .5 \frac{\text{m}}{\text{s}}$?

[11] Two stars (of masses $M_1 = 2 \times 10^{30}\text{ kg}$ and $M_2 = 3 \times 10^{30}\text{ kg}$ are initially at rest, at

a distance of $4 \times 10^{12} \text{ m}$ apart. Gravity will start pulling them towards each other. What is the speed of each star when the distance is one quarter of the initial distance?

REVIEW PROBLEMS: CHS. 6-11

① a) $K_i + U_i = K_f + U_f$ ($i = \text{INITIAL}$, $f = \text{FINAL}$)

$$0 + \frac{1}{2} kx^2 = \frac{1}{2} m v^2 + 0 \rightarrow v = \sqrt{\frac{kx^2}{m}} = \boxed{.98 \frac{m}{s}}$$

b) $F = k|x| = (1.5 \frac{N}{m})(1.08 m) = \boxed{.12 N}$ (MAGNITUDE)
(DIRECTION IS OPPOSITE TO $\vec{x}$)

② a) $K_i + U_i = K_f + U_f$

$$\frac{1}{2} m v_0^2 + 0 = \frac{1}{2} m v^2 + mgy \Rightarrow \frac{1}{2} v_0^2 = \frac{1}{2} v^2 + gy$$

SOLVE FOR FINAL v : $v = \sqrt{v_0^2 - 2gy} = \sqrt{(9)^2 - 2(9.8)(2)} = \boxed{6.47 \frac{m}{s}}$

b) $K_i + U_i = K_f + U_f$ ($v=0$ BOTH INITIALLY & FINALLY)

$$0 + \frac{1}{2} kx^2 = 0 + mgy \rightarrow \text{SOLVE FOR } x:$$

$$x = \sqrt{\frac{2mgy}{k}} = \sqrt{\frac{2(.2)(9.8)(4)}{10}} = \boxed{1.25 m}$$

③ a) $F = \frac{\Delta p_{ion}}{\Delta t} = \frac{\Delta(mv)_{ion}}{\Delta t} = v_{ion} \frac{\Delta m}{\Delta t} = (3 \times 10^6 \frac{m}{s})(10 \frac{kg}{s})$
 $= \boxed{3 \times 10^7 N}$

b) $a = \frac{F}{m} = \frac{3 \times 10^7 N}{2 \times 10^5 kg} = 1.5 \times 10^2 \frac{m}{s^2} = \boxed{150 \frac{m}{s^2}}$

$$v = v_0 + at = 0 + (150)(2) = \boxed{300 \frac{m}{s}}$$

MOMENTUM CONSERVATION

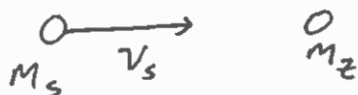
($s = \text{SHUTTLE}$)
($z = \text{ZRRGF}$)

c) $p_{\text{INITIAL}} = p_{\text{FINAL}} \Rightarrow m_s v_s = (m_s + m_z) v'$

$$v' = \frac{m_s v_s}{m_s + m_z} = \frac{(2 \times 10^5)(10^4)}{2 \times 10^5 + 4 \times 10^4} = \boxed{8333.3 \frac{m}{s}}$$

PART d
NEXT PAGE $\rightarrow$

d) INITIAL:



FINAL:



ONE-DIMENSION ELASTIC COLLISION $\rightarrow$ CAN USE SHORTCUT

MOMENTUM CONS.: $m_s v_s = m_s v'_s + m_z v'_z$ (A)

SHORTCUT: $v_s + v'_s = v'_z$ (B)

PLUG (B) INTO (A):

$$\begin{aligned} m_s v_s &= m_s v'_s + m_z (v_s + v'_s) \rightarrow (m_s - m_z) v_s = (m_s + m_z) v'_s \\ v'_s &= \left(\frac{m_s - m_z}{m_s + m_z} \right) v_s = \left(\frac{2 \times 10^5 - 4 \times 10^4}{2 \times 10^5 + 4 \times 10^4} \right) 10^4 \\ &= \boxed{+6.67 \times 10^3 \frac{\text{m}}{\text{s}}} \quad (\text{TO LEFT}) \end{aligned}$$

USE (B): $v'_z = v_s + v'_s = 10^4 + 6.67 \times 10^3$

$$= \boxed{1.67 \times 10^4 \frac{\text{m}}{\text{s}}}$$

(COULD ALSO USE ENERGY CONSERVATION INSTEAD OF (B). ALGEBRA IS MESSIER, BUT END RESULT IS THE SAME.)

Problem 4

$$\text{IMPULSE: } \Delta p = m v_f - m v_i = m v_f \quad (\text{SINCE } v_i = 0)$$

↑
m = MASS OF NAIL

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{m v_f}{\Delta t} = \frac{(0.005 \text{ kg})(15 \frac{\text{m}}{\text{s}})}{0.002 \text{ s}} = \boxed{37.5 \text{ N}}$$

↑
AVERAGE FORCE (BAR = AVERAGE)

$$\textcircled{5} \text{ a) } I = \sum m r^2 = m_1 r_1^2 + m_2 r_2^2 = (2)(2)^2 + (2)(2)^2 = \boxed{16 \text{ kg m}^2}$$

$$\text{b) } \sum \tau = \tau_1 + \tau_2 = 2\tau_1 = 2 F r \sin 30^\circ = 2(1.5 \text{ N})(2 \text{ m}) \sin 30^\circ$$

↑
(THE TWO FORCES EXERT EQUAL TORQUES)

$$= \boxed{3 \text{ N}\cdot\text{m}}$$

$$\sum \tau = I \alpha \Rightarrow \alpha = \frac{\sum \tau}{I} = \frac{3 \text{ N}\cdot\text{m}}{16 \text{ kg m}^2} = \boxed{.1875 \frac{\text{RAD}}{\text{s}^2}}$$

$$\text{c) } \omega = \omega_0 + \alpha t = 0 + (.1875 \frac{\text{RAD}}{\text{s}^2})(3 \text{ s}) = \boxed{.5625 \frac{\text{RAD}}{\text{s}}}$$

$$v = r \omega = (2 \text{ m})(.5625 \frac{\text{RAD}}{\text{s}}) = \boxed{1.125 \text{ m}}$$

⑥

a) $\omega_0 = 10 \frac{\text{RAD}}{\text{s}}$ $\omega_f = 0$ $\Delta t = 20 \text{ s}$

AVERAGE ANGULAR ACCELERATION: $\alpha = \frac{\Delta \omega}{\Delta t} = \frac{\omega_f - \omega_0}{\Delta t} = -0.5 \frac{\text{RAD}}{\text{s}^2}$

FROM TABLE IN BOOK: $I = \frac{1}{2} M r^2 = \frac{1}{2} (12 \text{ kg}) (0.8 \text{ m})^2 = 3.84 \text{ kg} \cdot \text{m}^2$

TORQUE: $\tau = I \alpha = (3.84 \text{ kg} \cdot \text{m}^2) (-0.5 \frac{\text{RAD}}{\text{s}^2}) = \boxed{-1.92 \text{ N} \cdot \text{m}}$

(THE MINUS SIGN GIVES DIRECTION)

b) $W = \Delta K + \Delta U \leftarrow \Delta U = 0$

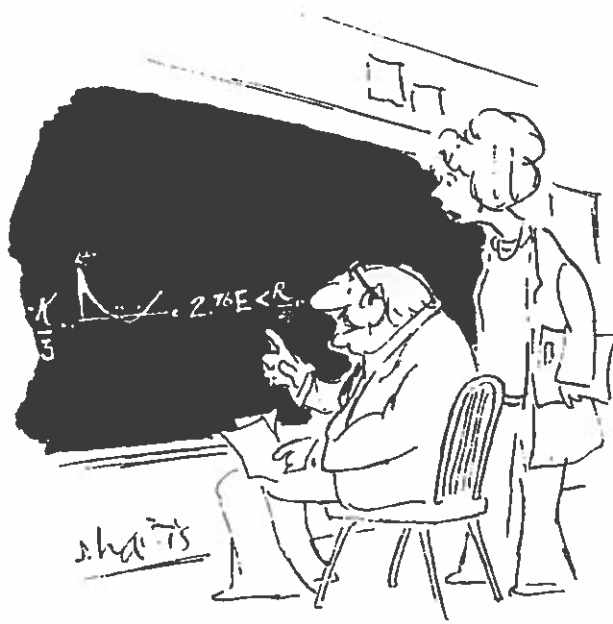
THE KINETIC ENERGY IS ALL ROTATIONAL:

$W = \Delta K = K_f - K_i = 0 - \frac{1}{2} I \omega_0^2$

$= -\frac{1}{2} (3.84) (10)^2 = \boxed{-192 \text{ J}}$

(THE MINUS SIGN MEANS THE CYLINDER LOST ENERGY.)

NEXT PAGE $\rightarrow$



"THE BEAUTY OF THIS IS THAT IT IS ONLY OF THEORETICAL IMPORTANCE, AND THERE IS NO WAY IT CAN BE OF ANY PRACTICAL USE WHATSOEVER."

⑦ $M = 3 \text{ kg}$, $R = .2 \text{ m}$, $y_i = 2 \text{ m}$, $y_f = 0$, $v_i = 0$.
 SPHERE: $I = \frac{2}{5} MR^2$

a) SLIDING WITH NO FRICTION. $\Rightarrow E_i = E_f$

$$U_i + \cancel{K_i} = \cancel{U_f} + K_f \rightarrow \cancel{Mg y_i} = \frac{1}{2} \cancel{M} v^2$$

$$g y_i = \frac{1}{2} v^2 \quad \leftarrow \quad K \text{ IS ALL TRANSLATIONAL.}$$

$$v = \sqrt{2g y_i} = \sqrt{2(9.8)(2)} = \boxed{6.26 \frac{\text{m}}{\text{s}}}$$

b) ROLLING WITHOUT SLIPPING $\Rightarrow$ ONLY STATIC FRICTION
 IS PRESENT $\Rightarrow$ STATIC FRICTION DOES NO WORK
 SO ONCE AGAIN E IS CONSERVED.

$$E_i = E_f \rightarrow U_i + \cancel{K_i} = \cancel{U_f} + K_f \rightarrow Mg y_i = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$$

NOW THERE IS ALSO $\uparrow$
ROTATIONAL KINETIC ENERGY.

USING $I = \frac{2}{5} MR^2$ AND $v = R\omega$, THE LAST EQUATION
 BECOMES:

$$Mg y_i = \frac{1}{2} M v^2 + \frac{1}{2} \left(\frac{2}{5} MR^2 \right) \left(\frac{v}{R} \right)^2$$

$$\quad \quad \quad \frac{1}{2} M v^2$$

$$\cancel{M} g y_i = \left(\frac{1}{2} + \frac{1}{5} \right) \cancel{M} v^2 \rightarrow g y_i = \frac{7}{10} v^2$$

$$v = \sqrt{\frac{10 g y_i}{7}} = \sqrt{\frac{10 (9.8) (2)}{7}} = \boxed{5.29 \frac{\text{m}}{\text{s}}}$$

IN PART a, ALL THE ENERGY WENT INTO v .

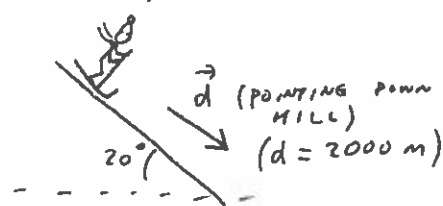
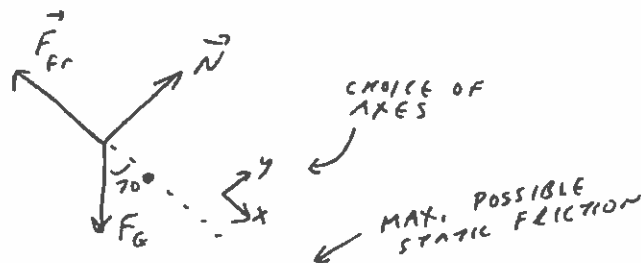
IN PART b, THE SAME AMOUNT OF ENERGY IS
 DIVIDED BETWEEN TWO DEGREES OF FREEDOM (v & ω)
 SO v IS SMALLER.

(THIS SHARING OF ENERGY BETWEEN TRANSLATIONAL & ROTATIONAL
 DEGREES OF FREEDOM WILL BE IMPORTANT FOR IDEAL GASES)

8

$$m = 60 \text{ kg}, \mu_k = .12, d = 2000 \text{ m}, \theta = 20^\circ,$$

a) FORCE DIAGRAM:



COMPONENTS:

$$\begin{cases} F_{frx} = -\mu_s N \\ F_{Gx} = mg \cos 70^\circ \\ N_x = 0 \\ a_x = a \geq 0 \end{cases}$$

$$F_{fry} = 0$$

$$F_{Gy} = -mg \sin 70^\circ$$

$$N_x = N \cos 20^\circ$$

$$a_y = 0$$

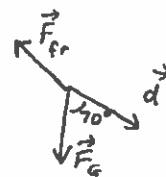
$$\sum F_y = ma_y \rightarrow -mg \sin 70^\circ + N = 0 \rightarrow \boxed{N = mg \sin 70^\circ}$$

$$\sum F_x = ma_x \rightarrow mg \cos 70^\circ - \mu_s N = ma_x \geq 0$$

$$\text{so } mg \cos 70^\circ \geq \mu_s N \quad \uparrow N = mg \sin 70^\circ$$

$$mg \cos 70^\circ \geq \mu_s mg \sin 70^\circ \rightarrow \mu_s \leq \frac{\cos 70^\circ}{\sin 70^\circ}$$

$$\text{so: } \mu_s \leq \cot 70^\circ = \tan 20^\circ = \boxed{.36}$$



b) NOW, FRICTION IS KINETIC:

$$F_{fr} = \mu_k N = \mu_k mg \sin 70^\circ = 66.3 \text{ N (AT } 180^\circ \text{ FROM } \vec{d})$$

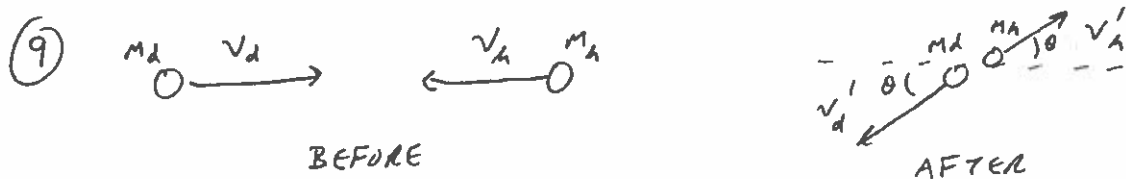
$$F_G = mg = (60)(9.8) = 588 \text{ N (AT } 70^\circ \text{ FROM } \vec{d})$$

SO WORKS ARE:

$$W_{fr} = F_{fr} d (\cos 180^\circ) = (66.3)(2000)(-1) = \boxed{-132600 \text{ J}}$$

$$W_G = F_G d (\cos 70^\circ) = (588)(2000)(.34) = \boxed{402215 \text{ J}}$$

$$\text{AT BOTTOM: } K = W_{fr} + W_G = 402215 - 132600 = \boxed{269615 \text{ J}}$$



GIVEN: $v_d = 2v_A = 2 \times 10^8 \frac{m}{s}$, $m_A = 2m_d$

i) MOMENTUM CONSERVATION: NO INITIAL y-MOMENTUM

$\sum p_y = \sum p_y' \Rightarrow 0 = -m_d v_d' \cos \theta + m_A v_A' \cos \theta$

SO: $v_d' = \frac{m_A}{m_d} v_A' = 2v_A' \rightarrow \boxed{v_d' = 2v_A'} \quad (*)$

(SINCE $m_A = 2m_d$)

$\sum p_x = \sum p_x' \Rightarrow m_d v_d - m_A v_A = -m_d v_d' \sin \theta + m_A v_A' \sin \theta$

USING $m_A = 2m_d$ AND $v_d' = 2v_A'$, THIS BECOMES:

$m_d v_d - 2m_d v_A = -m_d (2v_A') \sin \theta + (2m_d) v_A' \sin \theta$

CANCEL m_d FROM BOTH SIDES & SOLVE FOR θ :

$\underbrace{v_d - 2v_A}_{=0} = \underbrace{(-2v_A' + 2v_A') \sin \theta}_{=0} \Rightarrow 0 = 0$

NOT VERY INFORMATIVE

CAN'T FIND θ WITHOUT MORE INFORMATION.

ii) ENERGY CONSERVATION: $\sum K_i = \sum K_f$

$\frac{1}{2} m_d v_d^2 + \frac{1}{2} m_A v_A^2 = \frac{1}{2} m_d v_d'^2 + \frac{1}{2} m_A v_A'^2$

USING $m_A = 2m_d$, $v_d = 2v_A$, AND $v_d' = 2v_A'$:

$\underbrace{m_d (2v_A)^2 + (2m_d) v_A^2}_{6m_d v_A^2} = \underbrace{m_d (2v_A')^2 + (2m_d) v_A'^2}_{6m_d v_A'^2} \Rightarrow v_A^2 = v_A'^2$

SO $v_A = v_A' \rightarrow \boxed{v_A' = 10^8 \frac{m}{s}}$

BY EQUATION (*) ABOVE: $v_d' = 2v_A' = v_d = \boxed{2 \times 10^8 \frac{m}{s}}$

(10) $M_1 = 2 \times 10^{30} \text{ kg}$, $M_2 = 3 \times 10^{30} \text{ kg}$, $\underbrace{V_{10} = V_{20} = 0}_{\text{INITIAL}}$
 INITIALLY: $r_0 = 4 \times 10^{12} \text{ m}$

FINAL: $r = \frac{1}{4} r_0 = 10^{12} \text{ m}$

ENERGY CONSERVATION:

$$U_0 + K_0 = U + K \rightarrow -\frac{GM_1 M_2}{r_0} + 0 = -\frac{GM_1 M_2}{r} + \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 v_2^2$$

REARRANGE:

$$2GM_1 M_2 \left(\frac{1}{r} - \frac{1}{r_0} \right) = M_1 v_1^2 + M_2 v_2^2 \quad (A)$$

THE SYSTEM IS ISOLATED, SO MOMENTUM IS CONSERVED:

$$0 = M_1 v_1 - M_2 v_2,$$



SO: $v_2 = \frac{M_1 v_1}{M_2} \quad (B) \quad \leftarrow \text{PLUG INTO (A):}$

$$2GM_1 M_2 \left(\frac{1}{r} - \frac{1}{r_0} \right) = M_1 v_1^2 + M_2 \left(\frac{M_1 v_1}{M_2} \right)^2 = M_1 v_1^2 + \frac{M_1^2}{M_2} v_1^2$$

MULTIPLY BOTH SIDES BY $\frac{M_2}{M_1}$:

$$2GM_2^2 \left(\frac{1}{r} - \frac{1}{r_0} \right) = (M_2 + M_1) v_1^2$$

$$v_1 = \sqrt{\frac{2GM_2^2}{M_2 + M_1} \left(\frac{1}{r} - \frac{1}{r_0} \right)} = \sqrt{\frac{2(6.67 \times 10^{-11})(3 \times 10^{30})^2}{5 \times 10^{30}} \left(\frac{1}{10^{12}} - \frac{1}{4 \times 10^{12}} \right)}$$

$$= \boxed{1.34 \times 10^4 \frac{\text{m}}{\text{s}}}$$

FROM (B): $v_2 = \frac{M_1}{M_2} v_1 = \frac{2}{3} v_1 = \boxed{8.9 \times 10^3 \frac{\text{m}}{\text{s}}}$

⑪  $M_s = 2.2 \text{ kg}$ $v_s = .5 \frac{\text{m}}{\text{s}}$
 $v_w = 3 \frac{\text{m}}{\text{s}}$

RATE OF CHANGE OF MASS: $\begin{cases} \frac{dm_w}{dt} = .1 \frac{\text{kg}}{\text{s}} & (\text{WATER}) \\ \frac{dm_s}{dt} = -.1 \frac{\text{kg}}{\text{s}} & (\text{FISH}) \end{cases}$

a) $F = v_w \frac{dm_w}{dt} = (3)(.1) = \boxed{.3 \text{ N}}$

b) $F = \frac{dp}{dt} = \frac{d}{dt}(m_s v_s) = m_s \frac{dv_s}{dt} + v_s \frac{dm_s}{dt},$

so: $F = m_s a + v_s \frac{dm_s}{dt}$

$a = \frac{F - v_s \frac{dm_s}{dt}}{m_s} = \frac{.3 - (.5)(-.1)}{2.2}$

$= \boxed{.159 \frac{\text{m}}{\text{s}^2}}$

REVIEW PROBLEMS FOR FINAL

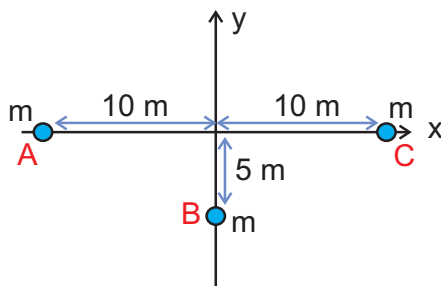
PHYSICS 121

[1] A satellite is in a circular orbit around the earth. It is in a geosynchronous orbit, meaning that it circles the earth once a day, and therefore always stays above the same point on the ground. (Geosynchronous orbits are usually used for communications satellites.) Find the radius r of the orbit (measured from the center of the earth), and the speed v of the satellite. (The mass of the earth is $M_E = 5.98 \times 10^{24} \text{ kg}$.)

[2] In the figure below, two point masses of size $m = 10^5 \text{ kg}$ are located at points A and C . These points are distances $x = 10 \text{ m}$ from the origin, on the x -axis. A third mass of the same size is located on the negative y -axis at position $y = -5 \text{ m}$ (point B).

(a) Find the total gravitational potential energy.

(b) Find the total gravitational force (magnitude and direction) on the mass at point B .



[3] Water (density $1000 \frac{\text{kg}}{\text{m}^3}$) flows down a river. When the river passes Jonestown, its depth is 10 m and the water flows at a speed of $.5 \frac{\text{m}}{\text{s}}$. When the river passes Smithtown, the river is twice as deep. (The width of the river stays the same.)

(a) What is the speed of the water as it passes Smithtown?

(b) At Jonestown, the gauge pressure at the river's bottom is 1.5 atm . What is the gauge pressure at the river bottom near Smithtown?

[4] After a mob hit, a suspicious-looking block of concrete is seen sinking into a New Jersey river. The block accelerates downward at $8.5 \frac{\text{m}}{\text{s}^2}$. After the block is recovered, it is found to have a mass of 1500 kg . What is the average density of the block? (The density of water is $1000 \frac{\text{kg}}{\text{m}^3}$.)

[5] (a) A rock is thrown straight upward with initial speed of $v_0 = 10 \frac{\text{m}}{\text{s}}$. How high does it go?

(b) A rocket is launched straight upward with initial speed of $v_0 = 10,000 \frac{\text{m}}{\text{s}}$. How high does it go? Why must part (b) be solved by a slightly different method than part (a)? (Useful information: the mass and radius of the earth are $5.98 \times 10^{24} \text{ kg}$ and $6.37 \times 10^6 \text{ m}$.)

[6] The densities of saltwater and of ice are $\rho_{sw} = 1024 \frac{kg}{m^3}$ and $\rho_{ice} = 917 \frac{kg}{m^3}$. Using this information, determine the fraction of an iceberg's total volume which will be above the surface of the water.

[7] A spherical helium-filled balloon has radius $R=12$ m. The balloon, support cables, and basket together have a mass of $m=196$ kg. (This does **not** include the mass of the helium.) What maximum cargo mass M can be carried in the basket, if the balloon is to remain afloat? (Use $\rho_{air} = 1.25 \frac{kg}{m^3}$ and $\rho_{He} = .16 \frac{kg}{m^3}$.)

[8] Water is pumped out of a storage tank 10 meters off the ground, through a pipe of .1m. The speed of the water when it leaves the tank is $.5 \frac{m}{s}$. The water emerges from a small hose of radius .05 m at ground level. The end of the hose is open to the air, so the pressure is 1 atm.

- What is the speed of the water as it emerges from the hose?
- What is the pressure in the tank?
- How long will it take to fill a smaller tank of volume $1000 m^3$?

[9] A 1.5 kg iron horseshoe initially at $600^\circ C$ is dropped into a bucket of .01 kg of water initially at $100^\circ C$. All of the water is turned into steam. Find the final temperature of the steam and horseshoe when the system finally reaches equilibrium. (Look up any information you need about the thermal properties of iron or water.)

[10] Pirates fire a cannonball at another ship, 100 m away. The initial speed of the cannonball is $40 \frac{m}{s}$.

- The pirate captain (a former physicist who found that crime pays much better) calculates the necessary angle at which the cannonball must be launched to hit the other ship. What answer does he find?
- What is the maximum height reached by the cannonball?

[11] An airplane wing is designed so that the speed of air over the top of the wing is $251 \frac{m}{s}$ when the speed of air under the wing is $225 \frac{m}{s}$. The density of air is $1.29 \frac{kg}{m^3}$. Find the net lift force on a wing of area $24.0 m^2$. (Assume that the height difference between the top and bottom of the wing is negligible.)

[12] An object of mass 12 kilograms sits on a level table. Someone pushes horizontally on it (toward the right) with a force of 6 Newtons. The coefficients of static and kinetic friction between the object and the table are $\mu_s = .15$ and $\mu_k = .12$.

- Does the object remain at rest or does it move? (Prove your answer.)
- If the object moves, find its acceleration. If the object stays at rest, find the size of the minimum push needed to make it move.

[13] An ideal gas initially has values $P_0 = 2 atm$, $T_0 = 300 K$, and $V_0 = .05 m^3$.

- If the gas is molecular nitrogen N_2 , what is the total mass of the gas?
- If the gas is raised to temperature $T = 400 K$ while keeping the volume fixed, what is the final pressure? What is the change in the internal energy of

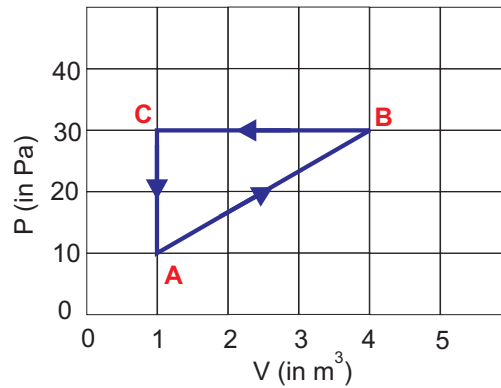
the gas? How much work was done and how much heat was added?

(c) Instead of the process of part b, suppose the gas of part (a) is compressed to half its initial volume, while holding the pressure fixed. What is the change in the internal energy of the gas? How much work was done and how much heat was added?

[14] Consider the process shown on the PV diagram below.

(a) Without assuming any knowledge of what the system consists of, find the work done and the change in internal energy of the system around the closed loop, assuming that 40 J of heat were added. (Note that the work will be negative if the process goes around the loop in the direction shown.)

(b) Now assume that the system consists of 5 moles of ideal gas. The process looks the same as before on the PV diagram, but now the heat is unknown. Find the work, the heat added to the system, and the change in internal energy of the system around the closed loop now.



REVIEW PROBLEMS (FOR FINAL EXAM)

SOLUTIONS

① PERIOD: $T = 1 \text{ DAY} = 86400 \text{ s}$

SPEED: $v = \frac{\text{DISTANCE}}{\text{TIME}} = \frac{2\pi r}{T}$ (A)

GRAVITY ACTS AS CENTRIPETAL FORCE:

$$F_g = ma_c \rightarrow G \frac{M_E M}{r^2} = \frac{mv^2}{r} \rightarrow \frac{GM_E}{r} = v^2 \quad (B)$$

COMBINE (A) & (B) TO ELIMINATE v :

$$\frac{GM_E}{r} = \left(\frac{2\pi r}{T}\right)^2 \rightarrow \text{SOLVE FOR } r:$$

$$r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67 \times 10^{-11}) (5.98 \times 10^{24}) (86400)^2}{4\pi^2}\right]^{1/3}$$

$$= \boxed{4.225 \times 10^7 \text{ m}}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi (4.225 \times 10^7)}{(86400)} = \boxed{3072.5 \frac{\text{m}}{\text{s}}}$$

② $r_{AB} = r_{BC} = \text{DISTANCE FROM A TO B, OR B TO C}$
 $= \sqrt{10^2 + 5^2} = \sqrt{125}$

$r_{AC} = \text{DISTANCE FROM A TO C} = 20 \text{ m}$

a) GRAVITATIONAL POTENTIAL ENERGY:

$$U = U_{AB} + U_{BC} + U_{AC} = -\frac{Gm^2}{r_{AB}} - \frac{Gm^2}{r_{BC}} - \frac{Gm^2}{r_{AC}}$$

$$= -Gm^2 \left(\frac{1}{r_{AB}} + \frac{1}{r_{BC}} + \frac{1}{r_{AC}} \right)$$

$$= -(6.67 \times 10^{-11}) (10^5)^2 \left(\frac{1}{\sqrt{125}} + \frac{1}{\sqrt{125}} + \frac{1}{20} \right)$$

$$= \boxed{-0.153 \text{ J}}$$

PART 6 ON?
 NEXT PAGE } $\rightarrow$

② CONTINUED

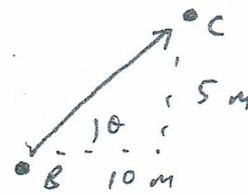
b) MAGNITUDE OF FORCE BETWEEN B & C:

$$F_{BC} = G \frac{m^2}{r_{BC}^2} = (6.67 \times 10^{-11}) \frac{(10^5)^2}{125} = 5.3 \times 10^{-3} \text{ N}$$

ANGLE:

$$\theta = \tan^{-1} \frac{5 \text{ m}}{10 \text{ m}} = 26.6^\circ$$

COMPONENTS:



$$F_{BCx} = (5.3 \times 10^{-3}) \cos 26.6^\circ = 4.74 \times 10^{-3} \text{ N}$$

$$F_{BCy} = (5.3 \times 10^{-3}) \sin 26.6^\circ = 2.37 \times 10^{-3} \text{ N}$$

THE FORCE OF A ON B WILL HAVE SAME MAGNITUDE, WITH COMPONENTS:

$$F_{AB,x} = -F_{BC,x} = -4.74 \times 10^{-3} \text{ N}$$

$$F_{AB,y} = +F_{BC,y} = +2.37 \times 10^{-3} \text{ N}$$

SO:

$$F_x = F_{ACx} + F_{BCx} = 0$$

$$F_y = F_{ACy} + F_{BCy} = 4.74 \times 10^{-3} \text{ N}$$

$$\text{MAGNITUDE: } F = \sqrt{F_x^2 + F_y^2} = 4.74 \times 10^{-3} \text{ N}$$

$$\text{DIRECTION: } \theta = \tan^{-1} \left(\frac{F_y}{F_x} \right) = 90^\circ$$

③ $v_J = .5 \frac{m}{s}$ WIDTHS: $w_S = w_J$
 DEPTH: $h_S = 2h_J$, $h_J = 10 \text{ m}$



a) $A_J v_J = A_S v_S \rightarrow w_J h_J v_J = w_S h_S v_S$

$w_J h_J v_J = (w_J) (2h_J) v_S$

$v_J = 2v_S \rightarrow v_S = \frac{1}{2} v_J = \boxed{.25 \frac{m}{s}}$

b) $P_J = 1.5 \text{ ATM} = 1.5 \times 10^5 \text{ Pa}$

$P_J + \rho g y_J + \frac{1}{2} \rho v_J^2 = P_S + \rho g y_S + \frac{1}{2} \rho v_S^2$

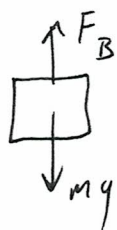
(NOTICE THAT $y = -h$)

$P_S = P_J + \rho g (y_J - y_S) + \frac{1}{2} \rho (v_J^2 - v_S^2)$

$= 1.5 \times 10^5 + (1000)(9.8)(-10 - (-20)) + \frac{1}{2}(1000)(.5^2 - .25^2)$

$= \boxed{2.48 \times 10^5 \text{ Pa}} \quad (= 2.46 \text{ ATM})$

④ $\sum F_y = ma_y \rightarrow F_B - mg = ma$ $a = -8.5 \frac{m}{s^2}$



$\rho_{H_2O} g V - mg = ma$

PLUG IN

BUT: $\rho_{BLOCK} = \frac{m}{V} \rightarrow V = \frac{m}{\rho_{BLOCK}}$

$\left(\frac{\rho_{H_2O}}{\rho_{BLOCK}} \right) mg - mg = ma \rightarrow \frac{\rho_{H_2O}}{\rho_{BL}} - 1 = \frac{a}{g}$

SOLVE:

$\rho_{BL} = \frac{\rho_{H_2O}}{\frac{a}{g} + 1} = \frac{1000}{\frac{-8.5}{9.8} + 1}$

$= \boxed{7538.5 \frac{kg}{m^3}}$

5) a) $v_0 = 10 \frac{m}{s}$. AT MAX HEIGHT: $v_f = 0$, so $K_f = 0$.

$$K_0 + U_0 = K_f + U_f \rightarrow \frac{1}{2} m v_0^2 + 0 = 0 + mgy$$

$$\text{SOLVE FOR } y: y = \frac{v_0^2}{2g} = \frac{(10)^2}{2 \cdot 9.8} = \boxed{5.1 \text{ m}}$$

b) $v_0 = 10000 \frac{m}{s}$, AGAIN, $v_f = K_f = 0$.

BUT NOW, BECAUSE v_0 IS MUCH LARGER, THE ROCKET WILL REACH A LARGE ENOUGH ALTITUDE THAT THE APPROXIMATION $U = mgy$ NO LONGER WORKS, MUST USE EXACT FORMULA

$$U = -\frac{GM_E m}{r}, \text{ WHERE } M_E = \text{MASS OF EARTH.}$$

INITIALLY, $r_0 = R_E$. AT HEIGHT y ABOVE THE EARTH'S SURFACE: $r_f = R_E + y$.

SO:

$$U_0 + K_0 = U_f + K_f$$

↓

$$-\frac{GM_E m}{R_E} + \frac{1}{2} m v_0^2 = -\frac{GM_E m}{R_E + y} + 0$$

SOLVE FOR y :

$$R_E + y = + \frac{GM_E}{\left(\frac{GM_E}{R_E} - \frac{1}{2} v_0^2\right)}$$

OR:

$$y = \frac{GM_E}{\frac{GM_E}{R_E} - \frac{1}{2} v_0^2} - R_E$$

$$\approx \left(\frac{(6.67 \times 10^{-11}) (5.98 \times 10^{24})}{6.37 \times 10^6 - \frac{(10000)^2}{2}} \right) - 6.37 \times 10^6$$

$$= \boxed{2.52 \times 10^7 \text{ m}} = 25,200 \text{ km}$$

- ⑥ LET $\begin{cases} V = \text{TOTAL VOLUME OF ICE BERG} \\ V_B = \text{VOLUME BELOW WATER} \end{cases}$

THEN WE WANT FRACTION $\frac{V - V_B}{V}$.

$\Sigma F = 0$, so $|\text{WEIGHT}| = |\text{BOUYANT FORCE}|$.

$$\underbrace{(\text{TOTAL MASS})}_{\rho_{\text{ice}} V} g = \underbrace{(\text{MASS OF DISPLACED WATER})}_{\rho_w V_B} g \rightarrow \rho_{\text{ice}} V = \rho_w V_B$$

SO, FRACTION BELOW WATER IS $\frac{V_B}{V} = \frac{\rho_{\text{ice}}}{\rho_w}$
AND FRACTION ABOVE IS:

$$\begin{aligned} \frac{V - V_B}{V} &= 1 - \frac{V_B}{V} = 1 - \frac{\rho_{\text{ice}}}{\rho_w} = 1 - \frac{917}{1024} \\ &= \boxed{.104} = 10.4\% \end{aligned}$$

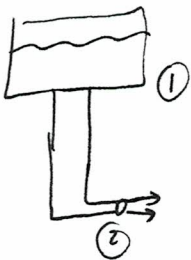
- ⑦ WEIGHT = BOUYANT FORCE:

$$\begin{array}{ccccccc} mg + \rho_{\text{He}} V g + Mg & = & \rho_{\text{air}} V g \\ \uparrow & \uparrow & \uparrow & \uparrow \\ \text{WEIGHT OF BALLOON} & \text{WEIGHT OF HELIUM} & \text{WEIGHT OF CARGO} & \text{WEIGHT OF DISPLACED AIR (BOUYANT FORCE)} \end{array}$$

SOLVE FOR M :

$$\begin{aligned} M &= \frac{(-m - \rho_{\text{He}} V + \rho_{\text{air}} V) / g}{g} = -m + V(\rho_{\text{air}} - \rho_{\text{He}}) \\ &= -m + \left(\frac{4}{3} \pi r^3\right)(\rho_{\text{air}} - \rho_{\text{He}}) \\ &= \boxed{7693.7 \text{ kg}} \quad \leftarrow (\text{VOLUME OF SPHERE}) \end{aligned}$$

8



POINT 1: $y_1 = 10 \text{ m}$, $r_1 = .1 \text{ m}$, $v_1 = .5 \frac{\text{m}}{\text{s}}$
 $A_1 = \pi r_1^2 = .0314 \text{ m}^2$

POINT 2: $P_2 = 1 \text{ ATM} = 1.01 \times 10^5 \text{ Pa}$
 $y_2 = 0$, $r_2 = .05 \text{ m}$
 $A_2 = \pi r_2^2 = 7.85 \times 10^{-3} \text{ m}^2$

a) $A_1 v_1 = A_2 v_2 \rightarrow v_2 = \frac{A_1}{A_2} v_1 = \left(\frac{.0314}{7.85 \times 10^{-3}} \right) (.5)$
 $= \boxed{2.0 \frac{\text{m}}{\text{s}}}$

b) $P_1 = P_2 + \rho g (y_2 - y_1) + \frac{1}{2} \rho (v_2^2 - v_1^2)$
 $= 1.01 \times 10^5 + (1000)(9.8)(-10) + \frac{1}{2} (1000)(2^2 - .5^2)$
 $= \boxed{4875 \text{ Pa}}$

c) $\text{VOLUME} = \left(\frac{\Delta V}{\Delta t} \right) \Delta t = A_1 v_1 \Delta t \quad (\text{or } A_2 v_2 \Delta t)$
 $\Delta t = \frac{\text{VOLUME}}{A_1 v_1} = \frac{1000}{(.0314)(.5)} = \boxed{6.37 \times 10^5 \text{ s}}$
 $(= 17.7 \text{ HOURS})$

9) $m_I = 1.5 \text{ kg}$, $T_I = 600^\circ \text{C}$, $m_W = .01 \text{ kg}$, $T_W = 100^\circ \text{C}$

$0 = Q_I + Q_W = \underbrace{m_I C_I (T - T_I)}_{Q_{\text{IRON}}} + \underbrace{[m_W C_S (T - 100^\circ \text{C}) + m_W L_v]}_{Q_{\text{WATER}}}$

SOLVE FOR T:

$T = \frac{m_I C_I T_I + m_W C_S (100^\circ \text{C}) - m_W L_v}{m_I C_I + m_W C_S}$

$= \frac{(1.5)(450)(600) + (.01)(4180)(100) - (.01)(6.09 \times 10^6)}{(1.5)(450) + (.01)(4180)}$

$= \boxed{485.9^\circ \text{C}}$

$\left(C_S = 2010 \frac{\text{J}}{\text{kg}^\circ \text{C}} (\text{STEAM}), C_I = 450 \frac{\text{J}}{\text{kg}^\circ \text{C}} (\text{IRON}) \right)$
 $L_v = 6.09 \times 10^6 \frac{\text{J}}{\text{kg}}$

⑩ a) $v_0 = 40 \frac{m}{s}$, $R = 100 \text{ m}$

$$R = \frac{v_0^2 \sin 2\theta}{g} \rightarrow \theta = \frac{1}{2} \sin^{-1} \left(\frac{Rg}{v_0^2} \right)$$

$$= \frac{1}{2} \sin^{-1} \left(\frac{(100)(9.8)}{(40)^2} \right) = \boxed{18.9^\circ}$$

b) $v_{0y} = v_0 \sin \theta = (40) \sin(18.9^\circ) = 12.95 \frac{m}{s}$

AT MAX. HEIGHT: $v_y = 0$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0) \rightarrow 0 = v_{0y}^2 - 2gy$$

$$y = \frac{v_{0y}^2}{2g} = \frac{(12.95)^2}{2(9.8)} = \boxed{8.56 \text{ m}}$$

⑪ $v_T = 251 \frac{m}{s}$, $v_B = 225$, $\rho = 1.29 \frac{kg}{m^3}$ $\left(\begin{matrix} T = \text{TOP} \\ B = \text{BOTTOM} \end{matrix} \right)$

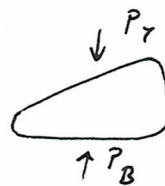
$A = 24 \text{ m}^2$, $y_T \approx y_B$

$$F_{LIFT} = (P_B - P_T) A$$

$$= \left[\frac{1}{2} \rho (v_T^2 - v_B^2) + \underbrace{\rho g (y - y_B)}_{\text{NEGLECTABLE}} \right] A$$

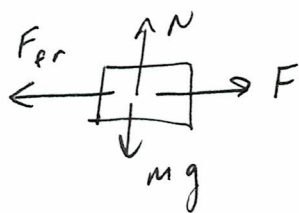
$$= \frac{1}{2} (1.29) (251^2 - 225^2) (24)$$

$$= \boxed{1.92 \times 10^5 \text{ N}}$$



(USING BERNOULLI)

(12) $m = 12 \text{ kg}$, $F = 6 \text{ N}$, $\mu_s = .15$, $\mu_k = .12$



a) $\sum F_y = ma_y \rightarrow N - mg = 0 \rightarrow N = mg$
 STATIC CASE:
 $F_{fr} \leq \mu_s N = \mu_s mg = 17.64 \text{ N}$

SINCE THE PUSH F IS LESS THAN THIS,
 THE BLOCK REMAINS **AT REST**.

b) THE MINIMUM PUSH WOULD EQUAL THE
 MAXIMUM STATIC FRICTION FORCE:
 $F = 17.64 \text{ N}$

(13) $P_0 = 2 \times 10^5 \text{ Pa}$, $T_0 = 300 \text{ K}$, $V_0 = .05 \text{ m}^3$

a) ATOMIC NITROGEN: $M_N = 14 \text{ g}$
 MOLECULAR NITROGEN: $M_{N_2} = 28 \text{ g}$

MOLES:

$$P_0 V_0 = n R T_0 \rightarrow n = \frac{P_0 V_0}{R T_0} = \frac{(2 \times 10^5)(.05)}{(8.314)(300)} = \boxed{4.0}$$

TOTAL MASS: $n M_{N_2} = (4)(28 \text{ g}) = \boxed{112 \text{ g}} (= .112 \text{ kg})$

b) $P V = n R T \rightarrow P = \frac{n R T}{V} = \frac{(4)(8.314)(400)}{.05} = \boxed{2.33 \times 10^5 \text{ Pa}}$

V HELD CONSTANT $\Rightarrow$ NO WORK: $\boxed{W = 0}$

DIATOMIC IDEAL GAS: $\Delta E = \frac{5}{2} n R \Delta T = \frac{5}{2} (4)(8.314)(100 \text{ K})$
 $= \boxed{8314 \text{ J}}$

$Q = \Delta E + W = 8314 + 0 = \boxed{8314 \text{ J}}$

$\rightarrow$
 PART C

c) NOW FINAL TEMP, IS: $T = \frac{PV}{nR} = \frac{(2 \times 10^5)(.025)}{4(8.314)} = 150 \text{ K}$

$$\Delta E = \frac{5}{2} n R \Delta T = \frac{5}{2} (4)(8.314) (-150 \text{ K}) = \boxed{12,472 \text{ J}}$$

$$W = P \Delta V = P (V_{\text{FINAL}} - V_{\text{INITIAL}})$$

$$= (2 \times 10^5 \text{ Pa})(.025 \text{ m}^3 - .05 \text{ m}^3) = \boxed{-5000 \text{ J}}$$

$$Q = \Delta E + W = 12,472 - 5,000 = \boxed{7,472 \text{ J}}$$

14 a) $W = -(\text{AREA ENCLOSED}) = -\frac{1}{2}(\text{BASE})(\text{HEIGHT})$

$$= -\frac{1}{2}(20 \text{ Pa})(3 \text{ m}^3) = \boxed{-30 \text{ J}}$$

(NEGATIVE BECAUSE IT IS GOING CLOCKWISE)

$$\Delta E = Q - W = 40 \text{ J} - (-30) = \boxed{70 \text{ J}}$$

b) $W = -30 \text{ J}$, AS BEFORE

NOW, $\boxed{\Delta E = 0}$ AROUND CLOSED LOOP (FOR IDEAL GAS).

SO: $Q = \Delta E + W = 0 + (-30) = \boxed{-30 \text{ J}}$

QUIZ #1

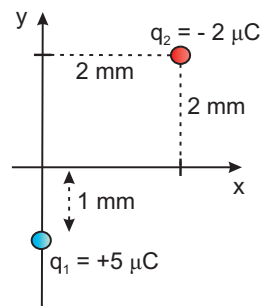
Physics II

Remember to show your work or give reasons for your answers.

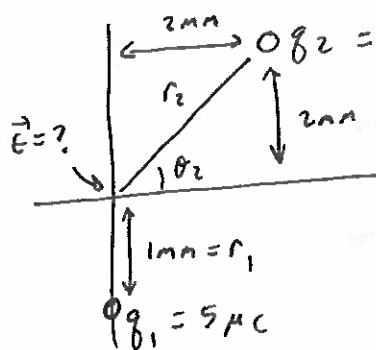
A pair of charges q_1 and q_2 is shown below. (Recall that $\mu = 10^{-6}$.)

(a) Find the magnitude and direction of the total electric field $\mathbf{E}$ that these charges produce at the origin.

(b) If an electron were to be placed at the origin, what would be the magnitude and direction of its acceleration? (The mass and charge of the electron are $m = 9.11 \times 10^{-31} \text{ kg}$ and $q = -1.6 \times 10^{-19} \text{ C}$.)



QUIZ #1 SOLUTIONS



$$\theta_1 = 90^\circ, \quad \theta_2 = \tan^{-1} \frac{2 \text{ mm}}{2 \text{ mm}} = 45^\circ$$

$$r_1 = 1 \text{ mm}$$

$$r_2 = \sqrt{(2 \text{ mm})^2 + (2 \text{ mm})^2} = 2.82 \text{ mm}$$

DISTANCES
FROM
ORIGIN

a) MAGNITUDES OF $\vec{E}_1$ & $\vec{E}_2$ AT ORIGIN:

$$E_1 = \left| \frac{kq_1}{r_1^2} \right| = \frac{(9 \times 10^9)(5 \times 10^{-6})}{(0.001 \text{ m})^2} = 4.5 \times 10^{10} \frac{\text{V}}{\text{m}}$$

$$E_2 = \left| \frac{kq_2}{r_2^2} \right| = \frac{(9 \times 10^9)(2 \times 10^{-6})}{(0.00282)^2} = 2.26 \times 10^9 \frac{\text{V}}{\text{m}}$$

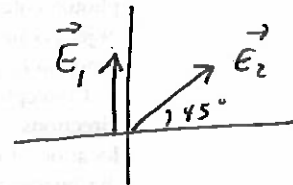
COMPONENTS:

$$E_{1y} = +E_1 = 4.5 \times 10^{10} \frac{\text{V}}{\text{m}}$$

$$E_{1x} = 0$$

$$E_{2x} = E_2 \cos \theta_2 = 1.6 \times 10^9 \frac{\text{V}}{\text{m}}$$

$$E_{2y} = E_2 \sin \theta_2 = 1.6 \times 10^9 \frac{\text{V}}{\text{m}}$$



TOTAL FIELD:

$$\begin{aligned} E_y &= E_{1y} + E_{2y} = 4.5 \times 10^{10} + 1.6 \times 10^9 = 4.66 \times 10^{10} \frac{\text{V}}{\text{m}} \\ E_x &= E_{1x} + E_{2x} = 0 + 1.6 \times 10^9 = 1.6 \times 10^9 \frac{\text{V}}{\text{m}} \end{aligned} \quad \left\{ \begin{array}{l} \text{OR -} \\ \vec{E} = (1.6 \hat{i} + 4.66 \hat{j}) \times 10^9 \frac{\text{V}}{\text{m}} \end{array} \right.$$

MAGNITUDE & DIRECTION:

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(1.6 \times 10^9)^2 + (4.66 \times 10^{10})^2} = \boxed{4.66 \times 10^{10} \frac{\text{V}}{\text{m}}}$$

$$\theta = \tan^{-1} \frac{E_y}{E_x} = \tan^{-1} \left(\frac{4.66 \times 10^{10}}{1.6 \times 10^9} \right) = \boxed{88^\circ}$$

$$\begin{aligned} \text{b) } \vec{F} &= q\vec{E} = m\vec{a} \Rightarrow a = \frac{qE}{m} = \frac{(1.6 \times 10^{-19} \text{ C})(4.66 \times 10^{10} \frac{\text{V}}{\text{m}})}{9.11 \times 10^{-31} \text{ kg}} \\ &= \boxed{8.2 \times 10^{21} \frac{\text{m}}{\text{s}^2}} \end{aligned}$$

DIRECTION IS OPPOSITE TO $\vec{E}$ (BECAUSE q IS NEGATIVE):

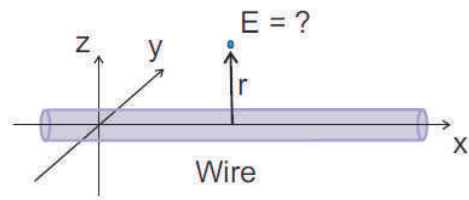
$$\theta = 88^\circ + 180 = \boxed{268^\circ}$$

QUIZ #2

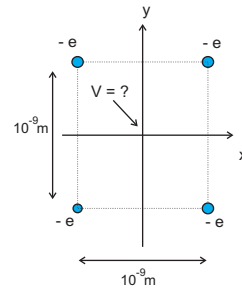
Physics II

Remember to show your work or give reasons for your answers.

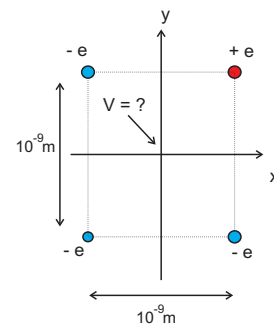
1. A very long cylindrical wire of radius R , stretched along the x -axis, has uniform ***volume*** charge density ρ (in $\frac{C}{m^3}$) embedded in it. Derive an expression for the magnitude of the electric field at a distance r from the center of the wire, **outside** the wire ($r > R$). Write the answer in terms of ρ , r , and R .



2. (a) Four electrons are placed at the corners of a square, as shown. Each side of the square is of length 10^{-9} m . Find the electric potential V of at the center of the square. (Assume $V = 0$ at infinity.)

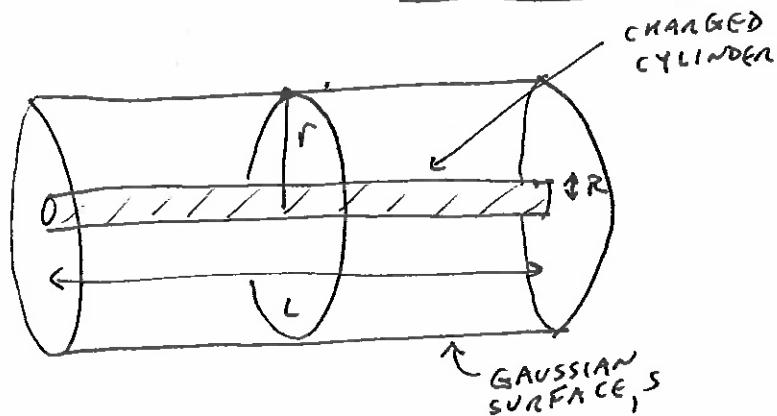


- (b) Suppose the electron at the top right corner of the square is replaced by a proton. What is the potential at the center of the square, now?



QUIZ #2 SOLUTIONS

①



$$r > R$$

$$\begin{aligned}\Phi_E &= \int_S \vec{E} \cdot d\vec{A} = EA \\ &= E(2\pi rL) \\ &\quad \text{SURFACE AREA OF GAUSSIAN CYLINDER, } S\end{aligned}$$

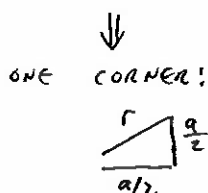
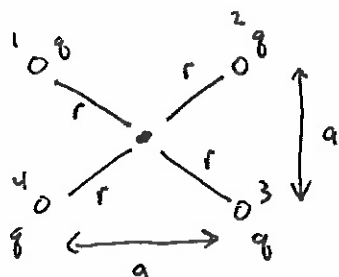
$$q_{enc} = \rho \cdot V = \rho(\pi R^2 L)$$

VOLUME OF CHARGE ENCLOSED

GAUSS: $\Phi_E = \frac{q_{enc}}{\epsilon_0} \rightarrow E(2\pi rL) = \rho(\pi R^2 L)/\epsilon_0$

$$E = \frac{\rho R^2}{2r\epsilon_0}$$

②



a) $a = 10^{-9} \text{ m}$, $q = -e = -1.6 \times 10^{-19} \text{ C}$

$$r = \frac{\sqrt{2}a}{2} = 7.07 \times 10^{-10} \text{ m}$$

$$V = \underbrace{V_1 + V_2 + V_3 + V_4}_{\text{ALL EQVM}} = 4V_1 = 4 \frac{kq}{r}$$

$$= 4 \left(\frac{kq}{\frac{\sqrt{2}a}{2}} \right) = \frac{4(9 \times 10^9)(-1.6 \times 10^{-19})}{7.07 \times 10^{-10}} = \boxed{-8.15 \text{ V}}$$

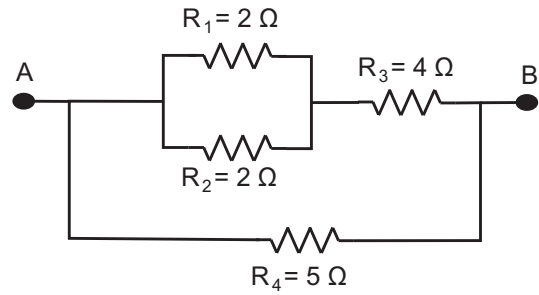
b) NOW $V_1 + V_3 + V_4 = -V_2 \rightarrow V = \overset{\text{CANCEL}}{\cancel{V_1} + \cancel{V_2} + V_3 + V_4} = \frac{2kq}{r}$

$$= \boxed{-4.07 \text{ V}}$$

QUIZ #3: Physics II

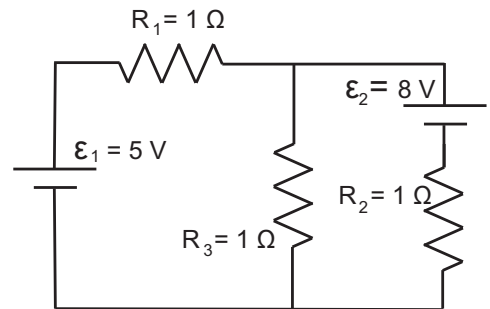
Remember to show your work.

(1) Find the total resistance between points A and B in the combination of resistors shown below.



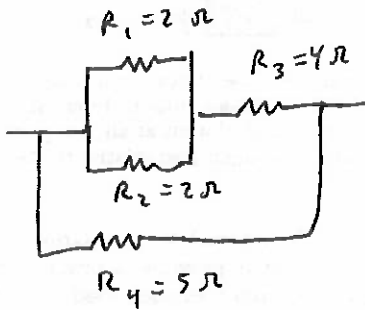
PROBLEM 2 ON BACK $\longrightarrow$

(2) Set up the equations that need to be solved for the all of the unknown currents in circuit below. (Don't try to solve the equations.) Clearly label the unknown currents on the picture, including the directions.

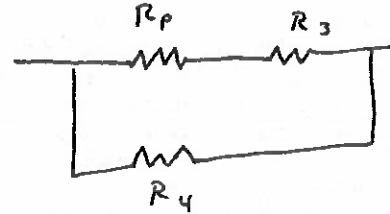


QUIZ 3 SOLUTIONS

①



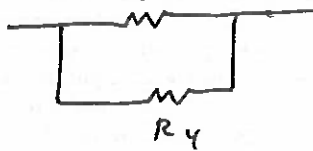
PARALLEL



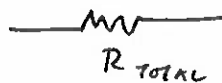
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{2} + \frac{1}{2} = 1 \Rightarrow R_p = 1\Omega$$

$$R_5 = R_p + R_3 = 1 + 4 = 5\Omega$$

SCALES



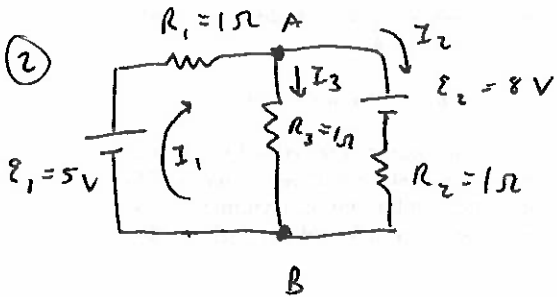
PARALLEL



$$\frac{1}{R_{\text{TOTAL}}} = \frac{1}{R_5} + \frac{1}{R_4} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

$$R_{\text{TOTAL}} = \frac{5}{2}\Omega = 2.5\Omega$$

②



2 JUNCTIONS → USE JUNCTION RULE ONCE:

$$I_1 = I_2 + I_3$$

USE LOOP RULE TWICE!

LEFT LOOP:

$$\mathcal{E}_1 - I_1 R_1 - I_3 R_3 = 0$$

$$(or \quad 5 - I_1 - I_3 = 0)$$

RIGHT LOOP:

$$+ I_3 R_3 - \mathcal{E}_2 - I_2 R_2 = 0$$

$$(or \quad 8 = I_3 - I_2)$$

- IF YOU CHOOSE TO DEFINE THE CURRENTS DIFFERENTLY, THE EQUATIONS WILL LOOK DIFFERENT, BUT WILL BE EQUIVALENT.

DO ALGEBRA. → SOLUTION!

$$I_1 = \frac{2}{3} A$$

$$I_2 = -\frac{11}{3} A$$

$$I_3 = \frac{13}{3} A$$

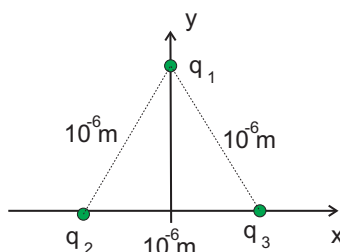
← $I_2 < 0 \Rightarrow I_2$ FLOWS OPPOSITE TO DIRECTION DRAWN.

REVIEW PROBLEMS
PHYSICS 122
Chapters 21-25

1. Consider the three charged particles shown below. They are at the corners of an equilateral triangle with side of length 10^{-6} m . Suppose that the particles are all electrons:

$$q_1 = q_2 = q_3 = -e.$$

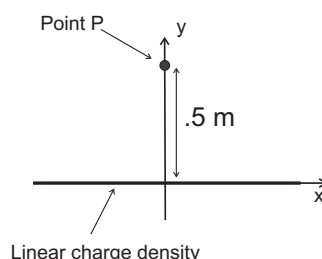
- (a) What is the magnitude and direction of the force on q_1 ?
- (b) What is the potential (voltage) at the location of q_1 ?
- (c) What is the potential energy of this configuration of charges?
- (d) Repeat part (a), but now assuming that q_2 is a proton instead of an electron: $q_2 = +e$.
- (e) Sketch the field lines for the situation of part (d).



2. A linear charge density of $10^{-10} \frac{\text{C}}{\text{m}}$ is distributed along an infinitely long line, lying along the x -axis.

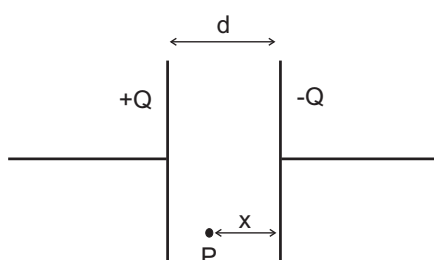
(a) Find the magnitude of the electric field at a point P on the y -axis, $.5 \text{ m}$ from the origin by using Gauss' law. (If you are up for a challenge, try obtaining the same result by integrating Coulomb's law over the charge distribution.)

(b) Find the magnitude and direction of the acceleration that an electron would experience if placed at point P .



3. Consider the parallel-plate capacitor shown below. The area of each plate is $A = 3 \times 10^{-4} \text{ m}^2$, the distance between the plates is 1.5 mm , and the capacitors is filled with air. The left plate has a positive charge of $3.5 \times 10^{-12} \text{ C}$ on it, with an equal and opposite charge on the right-hand plate.

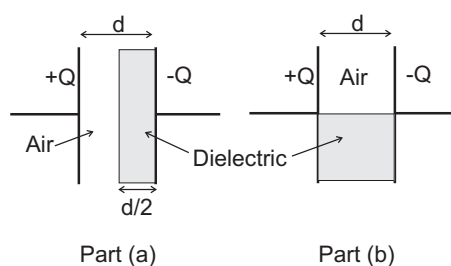
- Find the capacitance.
- Find the voltage difference between the plates.
- Find the magnitude and direction of the electric field inside the capacitor.
- If the voltage is taken to be zero at the negatively-charged plate, what is the potential at a point P 1 mm from the negative plate?



4. Consider again the capacitor from the previous problem.

(a) Suppose that right half of the capacitor is filled now with a dielectric of dielectric constant $\kappa = 3.5$, as shown in on left side of the figure below. The other half is still filled with air. What is the capacitance now?

(b) Suppose again that half of the capacitor is filled with dielectric, but now it is the bottom half, as shown on the right side of the figure. What is the new capacitance?



5. (a) Suppose the electric potential along the x -axis is given by the function $V(x) = ax^3 + b \ln x$, where a and b are constants. What is the electric field as a function of x ?

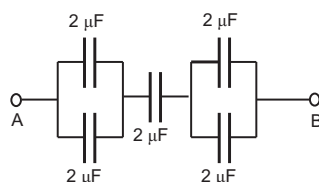
(b) Suppose now that the potential is given in the $x - y$ plane by $V(x, y) = ax^2y + by^2x$. What is the electric field as a function of x and y ?

6. Suppose a proton and an electron are held at rest a distance of 1 mm apart. If the electron is released, how fast will it be moving when it is 10^{-5} m from the proton?

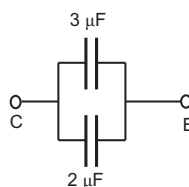
7. (a) Find the equivalent capacitance of each of the combinations below.

(b) Suppose a voltage of 10 V is applied between points A and B in the first combination of capacitors shown below. What is the charge on each plate of the capacitor in the middle? What is the voltage difference across that capacitor?

(c) Suppose a voltage of 10 V is applied between points C and D in the second combination of capacitors shown below. What is the charge on each of the two capacitors?

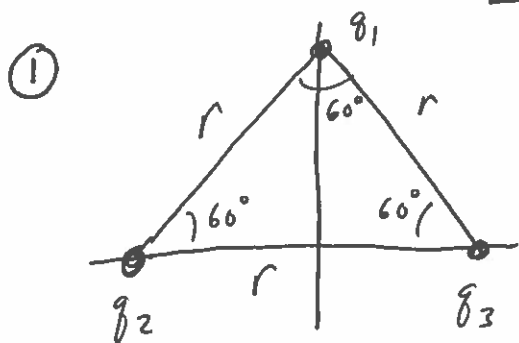


Parts (a) and (b)



Parts (a) and (c)

8. One more Gauss' law problem: Consider a charged sphere of radius R . Suppose that the volume charge density is *not* uniform, but instead is proportional to the distance r from the sphere's center: $\rho(r) = ar$, where a is a constant. Find (a) the total charge contained in the sphere (in terms of a), (b) the field inside the sphere, and (c) the field outside. (Hint: the volume of a sphere is $V = \frac{4}{3}\pi r^3$, so the corresponding differential (for doing integrals over volumes) is $dV = \frac{dV}{dr}dr = 4\pi r^2 dr$.)



$$q_1 = q_2 = q_3 = -e$$

$$r = 10^{-6} \text{ m}$$

a) Force of q_2 on q_1 :

$$F_{12} = \frac{k q_1 q_2}{r^2} = 9 \times 10^9 \frac{(-1.6 \times 10^{-19})^2}{(10^{-6})^2} = 2.3 \times 10^{-16} \text{ N}$$

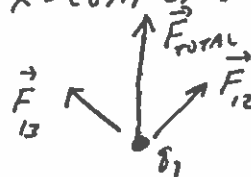
$$F_{12x} = F_{12} \cos \theta \quad \leftarrow \theta = 60^\circ = (2.3 \times 10^{-16}) \cos 60^\circ = 1.15 \times 10^{-16} \text{ N}$$

$$F_{12y} = F_{12} \sin \theta = (2.3 \times 10^{-16}) \sin 60^\circ = 1.99 \times 10^{-16} \text{ N}$$

Force of q_3 on q_1 is same except x-component changes sign:

$$F_{13x} = -1.15 \times 10^{-16} \text{ N}$$

$$F_{13y} = 1.99 \times 10^{-16} \text{ N}$$



so total force is:

$$F_x = F_{13x} + F_{12x} = 0$$

$$F_y = F_{13y} + F_{12y} = 3.98 \times 10^{-16} \text{ N}$$

MAGNITUDE: $3.98 \times 10^{-16} \text{ N}$
DIRECTION: UPWARD
(ALONG y-AXIS)

$$\begin{aligned} b) \quad V &= V_1 + V_2 = \frac{kq}{r} + \frac{kq_2}{r} = \frac{k}{r} (q_1 + q_2) \\ &= \frac{9 \times 10^9}{10^{-6}} (-1.6 \times 10^{-19} - 1.6 \times 10^{-19}) = \boxed{-1.0029 \text{ V}} \end{aligned}$$

$$\begin{aligned} c) \quad U &= U_{12} + U_{13} + U_{23} = \frac{k q_1 q_2}{r} + \frac{k q_1 q_3}{r} + \frac{k q_2 q_3}{r} \\ &= 3 \left(\frac{k q_1 q_2}{r} \right) = 3 \left(9 \times 10^9 \right) \frac{(-1.6 \times 10^{-19})^2}{10^{-6}} = \boxed{6.9 \times 10^{-22} \text{ J}} \end{aligned}$$

(SINCE ALL THREE OF THE TERMS ABOVE ARE EQUAL)

CONTINUED

(1) CONTINUED:

d) DIRECTION OF F_{12} FLIPS:
SO NOW!

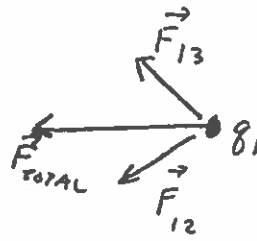
$$F_{12x} = -1.15 \times 10^{-16} \text{ N}$$

$$F_{12y} = -1.99 \times 10^{-16} \text{ N}$$

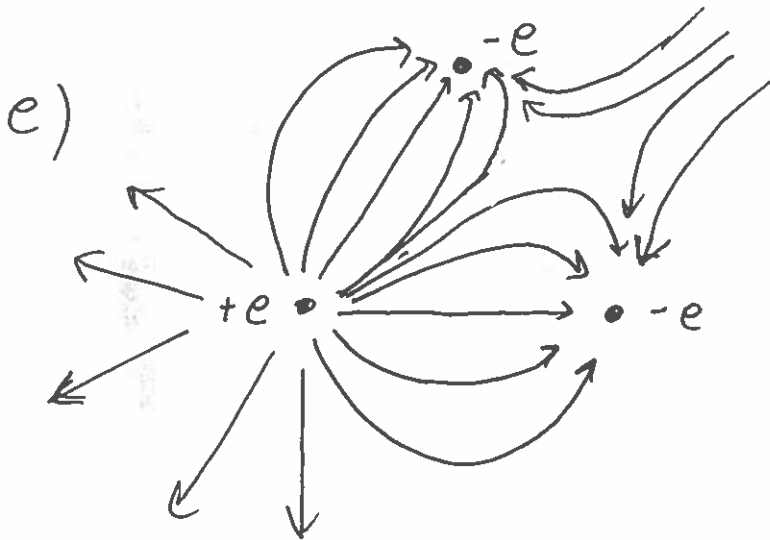
TOTAL FORCE:

$$F_x = F_{12x} + F_{13x} = -2.3 \times 10^{-16} \text{ N}$$

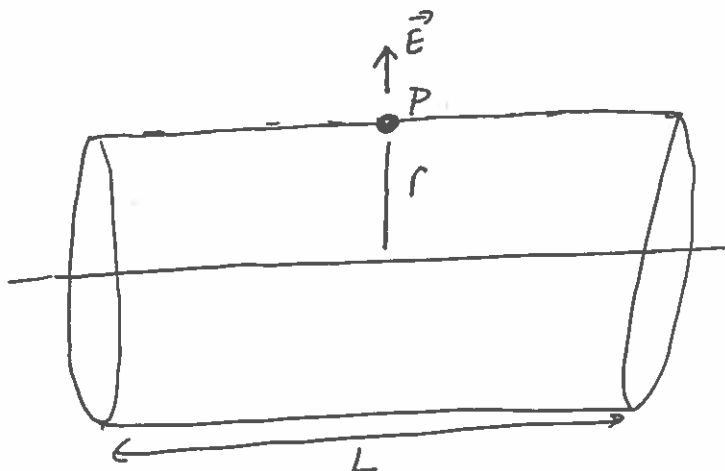
$$F_y = F_{12y} + F_{13y} = 0$$



MAGNITUDE:
 $F = 2.3 \times 10^{-16} \text{ N}$
DIRECTION:
NEGATIVE X-AXIS
(70°)



(2)



$$r = .5 \text{ m}$$

CHARGE PER LENGTH:

$$\lambda = 10^{-10} \text{ C/m}$$

- a) PICK AN ARBITRARY LENGTH L OF 'CHARGE', & CONSTRUCT GAUSSIAN CYLINDER THROUGH P AS SHOWN. FLUX THROUGH ENDS OF CYLINDER IS ZERO. FLUX THROUGH SIDES

$$\text{IS } \Phi = E \times (\text{AREA}) = 2\pi r L E.$$

BUT BY GAUSS' LAW, THE FLUX IS ALSO GIVEN BY:

$$\begin{aligned} \Phi &= \frac{1}{\epsilon_0} \times (\text{CHARGE ENCLOSED}) = \frac{1}{\epsilon_0} \left(\frac{\text{CHARGE PER LENGTH}}{\text{LENGTH}} \right) \times (\text{LENGTH}) \\ &= \frac{1}{\epsilon_0} \lambda L \end{aligned}$$

SET THESE TWO EXPRESSIONS EQUAL TO EACH OTHER:

$$2\pi r L E = \frac{1}{\epsilon_0} \lambda L \rightarrow E = \frac{\lambda}{2\pi r \epsilon_0} = \frac{10^{-10}}{(2\pi)(.5)(8.85 \times 10^{-12})}$$

$$= \boxed{3.6 \frac{\text{V}}{\text{m}}}$$

DIRECTION: OUTWARD, AWAY FROM POSITIVE CHARGE.

$$\begin{aligned} b) \quad \left. \begin{aligned} F &= m a \\ F &= q E \end{aligned} \right\} \rightarrow a &= \left| \frac{q E}{m} \right| = \frac{(1.6 \times 10^{-19})(3.6)}{9.11 \times 10^{-31}} \\ &= \boxed{6.3 \times 10^{11} \frac{\text{m}}{\text{s}^2}} \end{aligned}$$

INWARD, TOWARD POSITIVELY-CHARGED LINE.

$$\textcircled{3} \quad A = 3 \times 10^{-4} \text{ m}^2 \quad Q = 3.5 \times 10^{-12} \text{ C}$$

$$d = 1.5 \times 10^{-3} \text{ m} \quad x = 10^{-3} \text{ m}$$

$$a) \quad C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}) (3 \times 10^{-4})}{1.5 \times 10^{-3}} = \boxed{1.77 \times 10^{-12} \text{ F}}$$

$$(\quad = 1.77 \text{ pF})$$

$$b) \quad V = \frac{Q}{C} = \frac{3.5 \times 10^{-12}}{1.77 \times 10^{-12}} = \boxed{1.98 \text{ V}}$$

$$c) \quad E = \frac{V}{d} = \frac{1.98}{1.5 \times 10^{-3}} = \boxed{1320 \frac{\text{V}}{\text{m}}}$$

DIRECTION: TOWARD THE RIGHT (FROM +Q TOWARD -Q)

$$d) \quad (V \text{ AT POINT P}) = E \cdot x = \left(1320 \frac{\text{V}}{\text{m}}\right) \times 10^{-3} \text{ m}$$

$$= \boxed{1.32 \text{ V}}$$

$$\textcircled{4} \quad \kappa = 3.5 \rightarrow \epsilon = \kappa \epsilon_0 = (3.5)(8.85 \times 10^{-12}) = 30.98 \times 10^{-12} \frac{\text{F}}{\text{m}}$$

a) WE HAVE TWO CAPACITORS IN SERIES:

$$C_{\text{AIR}} = \frac{\epsilon_0 A}{d/2} = 3.54 \times 10^{-12} \text{ F}$$

$$C_{\text{DIELECTRIC}} = \frac{\epsilon A}{d/2} = \frac{(30.98 \times 10^{-12})(3 \times 10^{-4})}{.75 \times 10^{-3}} = 1.24 \times 10^{-11} \text{ F}$$

IN SERIES:

$$C = \frac{1}{C_{\text{AIR}}^{-1} + C_{\text{DIEL.}}^{-1}} = \frac{1}{(3.54 \times 10^{-12})^{-1} + (1.24 \times 10^{-11})^{-1}}$$

$$= 2.73 \times 10^{-12} \text{ F} = \boxed{2.73 \text{ pF}}$$

b) NOW HAVE TWO CAPACITORS IN PARALLEL:

$$C_{\text{AIR}} = \frac{\epsilon_0 (A/2)}{d} = 8.85 \times 10^{-13} \text{ F}$$

$$C_{\text{DIEL.}} = \frac{\epsilon (A/2)}{d} = 3.1 \times 10^{-12} \text{ F}$$

IN PARALLEL:

$$C = C_{\text{AIR}} + C_{\text{DIELECTRIC}} = 3.99 \times 10^{-12} \text{ F}$$

$$= \boxed{3.99 \text{ pF}}$$

⑤ a) $V(x) = ax^3 + b \ln x$

$$\vec{E}(x) = - \frac{dV}{dx} \hat{i} = \boxed{-\left(3ax^2 + \frac{b}{x}\right) \hat{i}}$$

(THE MINUS SIGN GIVES THE DIRECTION)

b) $V(x, y) = ax^2y - by^2x$

$$\vec{E}(x, y) = - \frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j}$$

$$= \boxed{(-2axy - by^2) \hat{i} + (-ax^2 - 2by) \hat{j}}$$

⑥ INITIAL & FINAL DISTANCES: $r_i = 10^{-3} \text{ m}$, $r_f = 10^{-5} \text{ m}$

$$K_i + U_i = K_f + U_f \rightarrow 0 + \frac{k(e)(-e)}{r_i} = \frac{1}{2}mv^2 + \frac{k(e)(-e)}{r_f}$$

$$\frac{1}{2}mv^2 = ke^2 \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

$$v = \sqrt{\frac{2ke^2}{m} \left(\frac{1}{r_f} - \frac{1}{r_i} \right)} = \sqrt{\frac{2(9 \times 10^9)(1.6 \times 10^{-19})^2}{9.11 \times 10^{-31}} \left(\frac{1}{10^{-5}} - \frac{1}{10^{-3}} \right)}$$

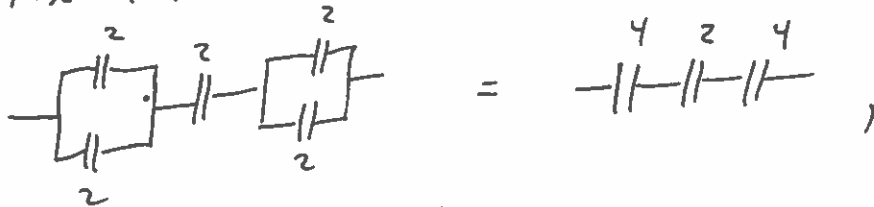
$$= \boxed{7071 \frac{\text{m}}{\text{s}}}$$

↑ ELECTRON MASS



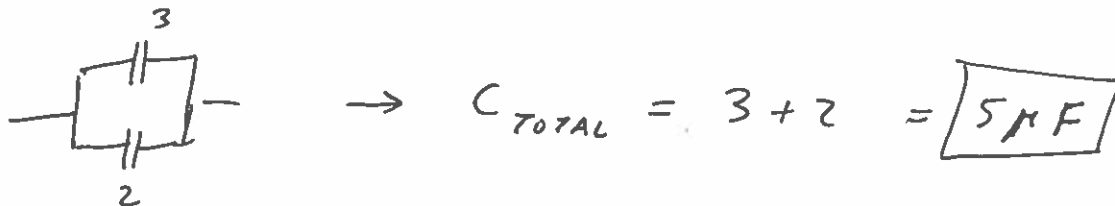
How teachers prepare for an upcoming school vacation.

⑦ a) FOR THE FIRST ONE:



so: $C = \frac{1}{\frac{1}{2} + \frac{1}{4} + \frac{1}{4}} = \boxed{1 \mu F}$

FOR THE SECOND:



b) THE TOTAL CAPACITANCE IS $C = 1 \mu C$ FROM ABOVE.
SO THE CHARGES AT THE TWO ENDS ARE

$$Q = \pm CV = \pm (1 \mu F)(10 V) = \pm 10^{-5} C$$

(THIS CHARGE IS SHARED BETWEEN THE 2 CAPACITORS)
IN EACH OF PARALLEL SECTIONS.

SINCE THE MIDDLE CAPACITOR IS IN SERIES WITH
THE PARALLEL SEGMENTS, IT WILL HAVE THE SAME
CHARGE:

$$\boxed{Q = \pm 10^{-5} C}$$

VOLTAGE ACROSS MIDDLE CAPACITOR:

$$V = \frac{Q}{C_{\text{MIDDLE}}} = \frac{10^{-5} C}{2 \times 10^{-6} F} = \boxed{5 V}$$

c) PARALLEL: BOTH CAPACITORS HAVE SAME VOLTAGE: $V = 10 V$.

TOP CAPACITOR: $Q = CV = (3 \times 10^{-6} F)(10 V) = \boxed{3 \times 10^{-5} C}$

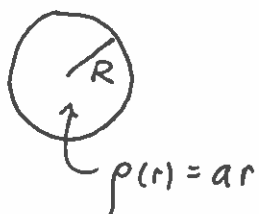
BOTTOM: $Q = CV = (2 \times 10^{-6} F)(10 V) = \boxed{2 \times 10^{-5} C}$

(NOTE THAT $Q_{\text{TOTAL}} = 5 \mu C$, SO THAT TOTAL CAPACITANCE

IS $C_{\text{TOT}} = \frac{Q_{\text{TOT}}}{V} = \frac{5 \times 10^{-5} C}{10 V} = 5 \mu F$, AGREEING WITH PART a.)

8

a) CHARGE:



$$\begin{aligned}
 Q &= \int \rho(r) dV = \int_0^R 4\pi r^2 (ar) dr \\
 &= 4\pi a \int_0^R r^3 dr = 4\pi a \left(\frac{R^4}{4} \right) \\
 &= \boxed{\pi a R^4}
 \end{aligned}$$

b) INSIDE SPHERE: $\leftarrow (r < R)$

$$Q_{ENC} = \int_0^r \rho(r) dV = \int_0^r 4\pi r^2 a r dr = \frac{4}{3} \pi a r^4$$

$$\oint \vec{E} \cdot d\vec{A} = E \underbrace{4\pi r^2}_{\text{SURFACE AREA OF SPHERE}}$$



SINCE $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{ENC}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \pi a r^4 / \epsilon_0$

$$\Downarrow$$

$$\boxed{E = \frac{a}{4\epsilon_0} r^2} \quad (\text{INSIDE } (r < R))$$

c) OUTSIDE ($r > R$):

$$\begin{aligned}
 Q_{ENC} &= \text{ENTIRE CHARGE OF SPHERE} \\
 &\quad (\text{FROM PART a}) \\
 &= \pi a R^4
 \end{aligned}$$

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) \longrightarrow E(4\pi r^2) = \frac{\pi a R^4}{\epsilon_0} \quad (\text{BY GAUSS})$$

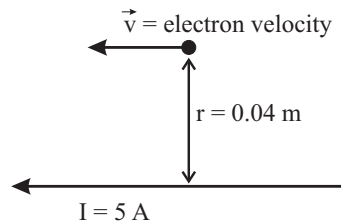
SOLVE FOR E :

$$\boxed{E = \frac{a R^4}{4\epsilon_0 r^2}}$$

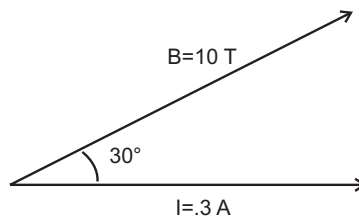
REVIEW PROBLEMS
PHYSICS 122
Chapters 26-30

1. An electron is traveling to the left with speed $3 \times 10^6 \frac{m}{s}$. The wire is carrying a current of 5 A. At the moment when the electron passes over the wire, .04 m away from it, find the magnitude and direction of:

- (a) the magnetic field of the wire at the electron's position.
- (b) the magnetic force on the electron. (Remember that the electron's charge is negative.)

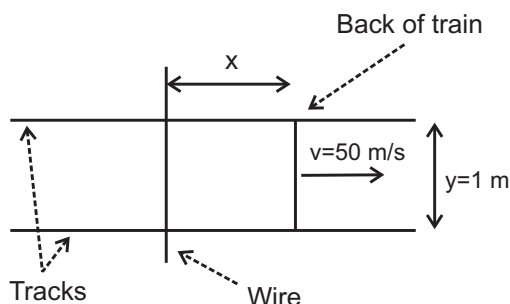


2. A straight wire of length .4 m is carrying a current of .3 A. There is a constant, uniform magnetic field of 10 T, at 30° to the wire, as shown. (a) Find the magnitude and direction of the magnetic force on the wire. (b) Suppose the wire is bent into the shape of a square. What is the magnetic flux through the loop? (The square is still at 30° to the field.) (c) If the magnetic field starts decreasing at a constant rate of $.2 \frac{T}{s}$, find the size of the induced EMF in the loop. (d) If the wire has a resistance of 1Ω , find the size and direction of the induced current caused by the EMF of part c.



3. A train is traveling at a speed of $50 \frac{m}{s}$, toward the east. The tracks are separated by a distance of 1 meter. A fallen wire lands on the tracks, making a closed circuit along with the tracks and the back of the train. The resistance of the circuit at a given moment is 100Ω . If the earth's magnetic field is about $6 \times 10^{-6} T$ (at 10° above the horizon), find the size of

the current induced by the train's motion through the field, and sketch its direction on the diagram.

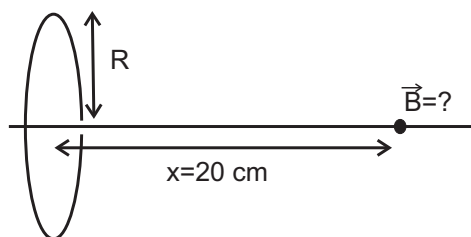


4. An electron is travelling at a speed of $3000 \frac{m}{s}$, perpendicular to the axis of a solenoid. The solenoid is .03 meters long and has 100 turns of wire. (a) If the solenoid is carrying 10 A of current, find the magnitude of the force on the electron. (b) If the electron is moving parallel to the axis instead of perpendicular to it, what would be the magnitude of the force on it?

5. A particle of charge $5 \times 10^{-16} C$ is traveling at constant speed of $2 \times 10^6 \frac{m}{s}$, perpendicular to a constant magnetic field of 20 T. Once it enters the field, the particle's path is bent into a circle. If the particle has mass $2 \times 10^{-28} kg$, what is the radius of this circle?

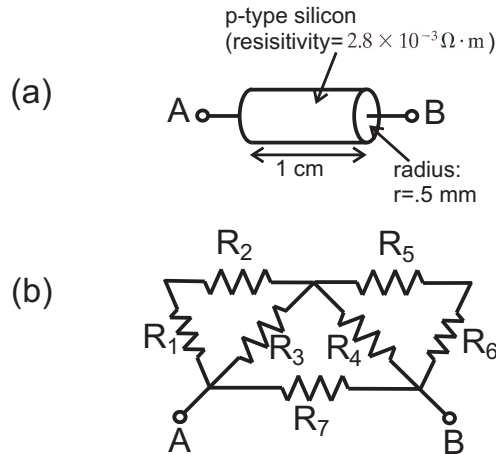
6. A wire has a current density that varies proportional to the distance from the axis: $J = br$ for $0 \leq r \leq R$. (Here, b is a constant and R is the radius of the wire.) For both points inside ($a < R$) and outside ($a > R$) the wire, find an expression for the magnetic field at a point distance a from the center. Give the answer as a function of the variables given (b , R , and a).

7. A single loop of wire (radius 2 cm) carries a current of .65 A. Find the magnetic field $\vec{B}$ at a distance of 20 cm from the center of the loop.



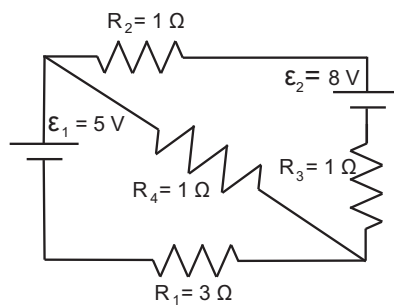
8. A solenoid has mean radius r , cross-sectional area A , and number of loops N_1 . A second solenoid is wrapped around the first, with number of loops N_2 . Find the mutual inductance. (Neglect the variation of the magnetic field across the cross section of the solenoid.) Assume that both coils are of the same length l and that the outer coil is tightly wound around the first so that it has approximately the same area, A .

9. Find the total resistance between points A and B in each of the following situations. (In part b, all of the resistors are 2Ω .)

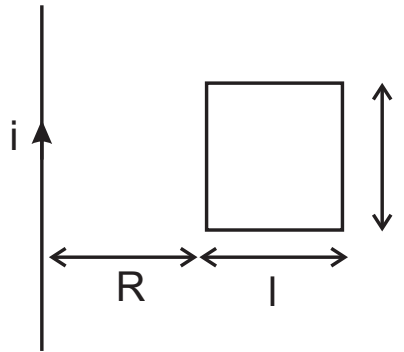


10. (a) Find the magnitude and direction of all of the unknown currents in the circuit below.

(b) Find the rate at which heat is generated in each of the resistors.



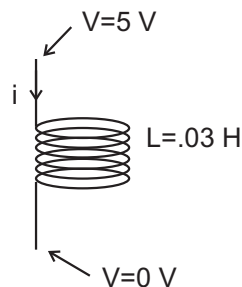
11. Consider a straight wire sitting near a single square loop of wire, as shown. Take all the dimensions as shown, and assume the current in the straight wire is parallel to one side of the square loop. Find the mutual inductance of the straight wire and the loop. (Hint: you will need to do an integral.)



12. Suppose an inductor in an electric circuit has a voltage difference of $\mathcal{E} = 5 \text{ V}$ across it, as shown. The inductance is $L = 30 \text{ mH} = .03 \text{ H}$.

(a) How fast is the current in the circuit changing? (Don't worry here about whether the change is positive or negative.)

(b) If the top of the inductor is at the higher voltage and the current is traveling downward, as shown, then is the current increasing in size or decreasing?



① $v = 3 \times 10^6 \frac{m}{s}$ (TO LEFT), $I = 5 A$ (TO LEFT)
 $r = .04 m$

a) $B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} I}{2\pi r} = \frac{(2 \times 10^{-7})(5)}{.04}$
 $= \boxed{2.5 \times 10^{-5} T}$ (INTO PAGE)

b) $|\vec{F}| = |q \vec{v} \times \vec{B}| = q v B \sin \theta$ ← BETWEEN $\vec{v}$ & $\vec{B}$:
 $\theta = 90^\circ$
 $= (1.6 \times 10^{-19})(3 \times 10^6)(2.5 \times 10^{-5})(1)$
 $= \boxed{1.2 \times 10^{-17} N}$ (UP) ← $\vec{v} \times \vec{B}$ IS DOWN,
BUT CHARGE IS
NEGATIVE

② $l = .4 m$, $I = .3 A$, $B = 10 T$, $\theta = 30^\circ$ BETWEEN $\vec{l}$ & $\vec{B}$.

a) $F = |I \vec{l} \times \vec{B}| = (.3)(.4)(10) \sin 30^\circ = \boxed{.6 N}$
(OUT OF PAGE)

b) $A = \left(\frac{l}{4}\right)^2 = .01 m^2 \rightarrow \mu = I A = .003 A m^2$ (UPWARD)

FLUX: $\Phi_B = \vec{B} \cdot \vec{A} = (10)(.01) \cos 30^\circ = \boxed{.087 Wb}$

c) $\frac{dB}{dt} = -.2 \frac{T}{s}$

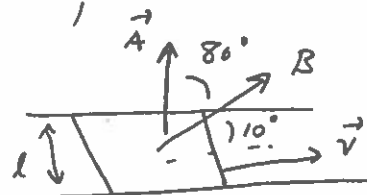
$|\mathcal{E}| = \left| N \frac{d\Phi_B}{dt} \right| = N \left| \frac{d\vec{B}}{dt} \cdot \vec{A} \right| = N \left| \frac{dB}{dt} \right| A \cos \theta = (1)(.2)(.01) \cos 30^\circ$
 $= \boxed{1.73 \times 10^{-3} V}$

d) $I = \frac{\mathcal{E}}{R} = \frac{1.73 \times 10^{-3} V}{1 \Omega} = \boxed{1.73 \times 10^{-3} A}$

COUNTERCLOCKWISE AS VIEWED FROM ABOVE
(CLOCKWISE AS VIEWED FROM BELOW)

③ $v = 50 \frac{m}{s}$ to right, $l = 1m$, $R = 100 \Omega$

$B = 6 \times 10^{-6} T$, $\theta = 10^\circ$



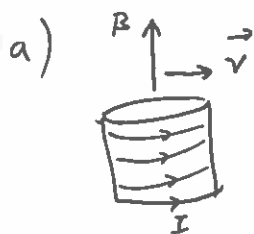
$$\mathcal{E} = N \left| \frac{d\Phi_B}{dt} \right| = \left| N \left(\frac{dA}{dt} \right) B \cos \theta \right|$$

$$= \left| N B l \underbrace{\frac{dx}{dt}}_v \cos \theta \right| = (1)(6 \times 10^{-6})(1)(50) \cos 80^\circ$$

$$= \boxed{5.2 \times 10^{-5} V} \quad \left(\begin{array}{c} \text{BETWEEN} \\ \vec{B} \text{ \& } \vec{A} \end{array} \right)$$

$$I = \frac{\mathcal{E}}{R} = \boxed{5.2 \times 10^{-7} A} \rightarrow \text{CLOCKWISE AS VIEWED FROM ABOVE.}$$

④ $q = -e$, $v = 3000 \frac{m}{s}$, $l = .03 m$, $N = 100$, $I = 10 A$



$$B = \mu_0 n I = (4\pi \times 10^{-7})(3333.3)(10) = \boxed{.042 T} \quad (UP)$$

$$n = \frac{N}{l} = \frac{100}{.03} = 3333.3 m^{-1}$$

$$\text{FORCE: } F = |q \vec{v} \times \vec{B}| = (1.6 \times 10^{-19})(3000)(.042) \times \sin 90^\circ = \boxed{2.02 \times 10^{-17} N}$$

INTO PAGE } USING FACT THAT q IS NEGATIVE

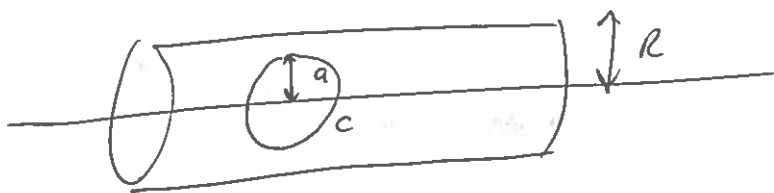
b) Now: $\boxed{\vec{F} = 0}$, BECAUSE $\vec{v}$ & $\vec{B}$ ARE PARALLEL (so $\vec{v} \times \vec{B} = 0$)

⑤ $q = 5 \times 10^{-16} C$, $v = 2 \times 10^6 \frac{m}{s}$, $B = 20 T$, $m = 2 \times 10^{-28} kg$

$$F = ma \rightarrow |q \vec{v} \times \vec{B}| = \frac{mv^2}{r} \rightarrow q v B \sin 90^\circ = \frac{mv^2}{r}$$

$$qB = \frac{mv}{r} \rightarrow r = \frac{mv}{qB} = \frac{(2 \times 10^{-28})(2 \times 10^6)}{(5 \times 10^{-16})(20)} = \boxed{4 \times 10^{-8} m}$$

⑥



$$J = br$$

CASE 1: OUTSIDE WIRE ($a > R$)

C ENCLOSES ENTIRE WIRE, SO $I_{ENC} = \text{TOTAL CURRENT}$:

$$I_{ENC} = \int J dA = \int_0^R br (2\pi r dr) \\ = 2\pi b \int_0^R r^2 dr = \frac{2}{3} \pi b R^3$$

AMPERES:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{ENC} \rightarrow B(2\pi a) = \mu_0 \left(\frac{2}{3} \pi b R^3 \right)$$

$$\boxed{B = \frac{\mu_0 b R^3}{3a}}$$

CASE 2: INSIDE WIRE ($a < R$)

ONLY CHANGE IS THAT NOT ALL THE CURRENT IS ENCLOSED INSIDE C. WE ONLY INTEGRATE UP TO a INSTEAD OF R !

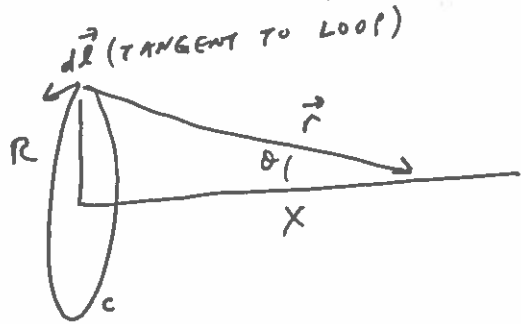
$$I_{ENC} = \int_0^a br (2\pi r dr) = 2\pi b \int_0^a r^2 dr = \frac{2\pi b}{3} a^3$$

so:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{ENC} \rightarrow B(2\pi a) = \mu_0 \left(\frac{2\pi b}{3} a^3 \right)$$

$$\boxed{B = \frac{\mu_0 b a^2}{3}}$$

⑦ $R = .02 \text{ m}$, $N = 1$, $I = .65 \text{ A}$, $x = .2 \text{ m}$



BY SYMMETRY ONLY X COMPONENT CAN BE NONZERO.

USE BIOT-SAVART.

$$r = \sqrt{x^2 + R^2}, \quad \sin \theta = \frac{R}{\sqrt{x^2 + R^2}}$$

$$\cos \theta = \frac{x}{\sqrt{x^2 + R^2}}$$

$$dB = \frac{\mu_0}{4\pi} \left| \frac{d\vec{l} \times \vec{r}}{r^3} \right| = \frac{\mu_0}{4\pi} \frac{(dl) r}{r^3} \sin 90^\circ$$

$$= \frac{\mu_0}{4\pi} \frac{dl}{r^2} = \frac{\mu_0}{4\pi} \left(\frac{dl}{x^2 + R^2} \right)$$

$d\vec{l} \perp \vec{r}$ SINCE $\vec{r}$ IN PLANE OF PAGE, $d\vec{l}$ OUT OF PAGE

$$dB_x = dB \cos \theta = \underbrace{\frac{\mu_0}{4\pi} \left(\frac{dl}{x^2 + R^2} \right)}_{dB} \underbrace{\left(\frac{x}{\sqrt{x^2 + R^2}} \right)}_{\cos \theta} = \frac{\mu_0 x I}{4\pi (x^2 + R^2)^{3/2}} dl$$

$$B_x = \int dB_x = \frac{\mu_0 x I}{4\pi (x^2 + R^2)^{3/2}} \underbrace{\int dl}_{\text{CIRCUMFERENCE}}$$

INTEGRATE AROUND WIRE

$$= \frac{\mu_0 x I}{4\pi (x^2 + R^2)^{3/2}} \cdot (2\pi R)$$

$$B_x = \frac{\mu_0 x R I}{2 (x^2 + R^2)^{3/2}}, \quad B_y = B_z = 0$$

⑧ FIELD FROM COIL 1: $B_1 = \mu_0 n_1 I_1 = \frac{\mu_0 N_1}{l} I_1$
 FLUX THROUGH 2nd COIL:

$$\Phi_{B_2} = \int B_1 dA_2 = B_1 A = \frac{\mu_0 N_1}{l} A I_1$$

$$M = N_2 \frac{d\Phi_{B_2}}{dI_1} = \boxed{\frac{\mu_0 N_1 N_2}{l} A}$$

⑨ a) $R = \frac{\rho l}{A} = \frac{(2.8 \times 10^{-3})(.01m)}{\pi(.005m)^2} = \boxed{.36 \Omega}$

b) R_1 & R_2 IN SERIES WITH TOTAL RESISTANCE:

$$R_A = R_1 + R_2 = 2 + 2 = 4 \Omega$$

R_5 & R_6 IN SERIES: $R_B = R_5 + R_6 = 4 \Omega$



R_A & R_3 IN

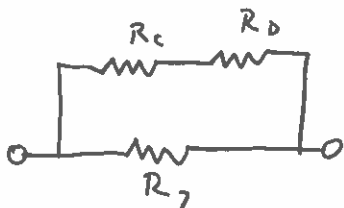
$$\frac{1}{R_C} = \frac{1}{R_A} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$R_C = \frac{4}{3} \Omega$$

SIMILARLY, R_B & R_4 IN PARALLEL:

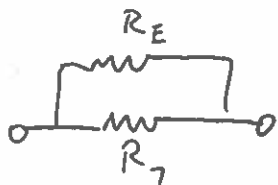
$$\frac{1}{R_D} = \frac{1}{R_B} + \frac{1}{R_4} \rightarrow R_D = \frac{4}{3} \Omega$$

SO:



ADD R_C & R_D IN SERIES:

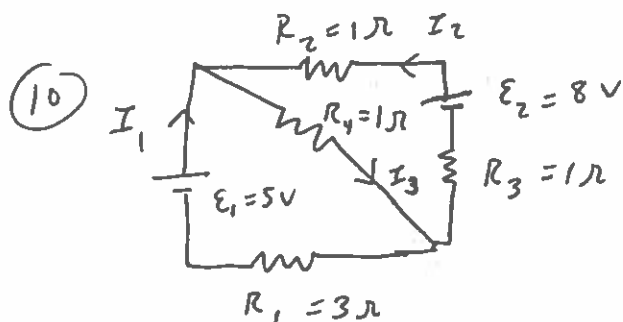
$$R_E = R_C + R_D = \frac{8}{3} \Omega$$



ADD R_E & R_7 IN PARALLEL:

$$\frac{1}{R} = \frac{1}{R_E} + \frac{1}{R_7} = \frac{3}{8} + \frac{1}{2} = \frac{7}{8}$$

$$\boxed{R = \frac{8}{7} \Omega} \quad (\approx 1.14 \Omega)$$



JUNCTION RULE:

$$I_1 + I_2 = I_3 \quad (A)$$

LOOP RULE:

$$E_1 - I_3 R_4 - I_1 R_1 = 0 \quad (B)$$

$$E_2 - I_2 R_2 - I_3 R_4 - R_3 I_2 = 0 \quad (C)$$

PLUG (A) INTO (C) TO ELIMINATE I_2 !

$$E_2 - I_3 (R_2 + R_3 + R_4) + I_1 (R_2 + R_3) = 0$$

SOLVE FOR I_1 :

$$I_1 = \frac{I_3 (R_2 + R_3 + R_4) - E_2}{R_2 + R_3} \quad (D)$$

PLUG THIS INTO (B) TO ELIMINATE I_1 !

$$E_1 - I_3 R_4 - \frac{R_1 (R_2 + R_3 + R_4) I_3 - E_2 R_1}{R_2 + R_3} = 0$$

SO:

$$I_3 \left[\frac{R_4 (R_2 + R_3) + R_1 (R_2 + R_3 + R_4)}{R_2 + R_3} \right] = E_1 - \frac{E_2 R_1}{R_2 + R_3}$$

SOLVE FOR I_3 :

$$I_3 = \frac{(R_2 + R_3) E_1 - E_2 R_1}{R_4 (R_2 + R_3) + R_1 (R_2 + R_3 + R_4)} = \boxed{\frac{34}{11} A \approx 3.09 A}$$

PLUGGING BACK INTO (A) & (D):

$$I_1 = \frac{E_1 (R_2 + R_3 + R_4) - E_2 R_4}{R_4 (R_2 + R_3) + R_1 (R_2 + R_3 + R_4)} = \boxed{\frac{7}{11} A \approx .636 A}$$

$$I_2 = \frac{E_2 (R_1 + R_4) - E_1 R_4}{R_4 (R_2 + R_3) + R_1 (R_2 + R_3 + R_4)} = \boxed{\frac{27}{11} A \approx 2.45 A}$$

- DIRECTIONS ARE AS DRAWN

⑪ AT DISTANCE r FROM THE WIRE THE FIELD IS: $B = \frac{\mu_0 i}{2\pi r}$ (INTO PAGE AT LOCATION OF WIRE LOOP)

FLUX THROUGH LOOP:

$$\begin{aligned}\Phi_B &= \int B \cdot dA = \int B \cos\theta dA \\ &\quad \cos\theta = 1 \\ &= \int_R^{R+l} \underbrace{\left(\frac{\mu_0 i}{2\pi r}\right)}_B \underbrace{l dr}_{dA} = \frac{\mu_0 i l}{2\pi} \int_R^{R+l} \frac{dr}{r} \\ &= \frac{\mu_0 i l}{2\pi} \ln\left(\frac{R+l}{R}\right)\end{aligned}$$

$dA = l dr$
AREA OF PORTION OF LOOP OF WIDTH dr

THERE IS ONLY ONE LOOP, SO:

$$M = N \frac{d\Phi_B}{di} = (1) \frac{d}{di} \left[\frac{\mu_0 i l}{2\pi} \ln\left(\frac{R+l}{R}\right) \right]$$

$$M = \frac{\mu_0 l}{2\pi} \ln\left(\frac{R+l}{R}\right)$$

⑫ $L = 30 \text{ mH} = .03 \text{ H}$, $\mathcal{E} = 5 \text{ V}$

a) $|\mathcal{E}| = \left| L \frac{dI}{dt} \right| \rightarrow \frac{dI}{dt} = \left| \frac{\mathcal{E}}{L} \right| = \frac{5}{.03} = \boxed{166.7 \frac{\text{A}}{\text{s}}}$

b) $\mathcal{E}$ OPPOSES $\frac{dI}{dt}$. SINCE $\mathcal{E}$ IS INCREASING FROM BOTTOM TO TOP I MUST BE CHANGING IN THE DOWNWARD DIRECTION. AS SHOWN IN THE PICTURE, THE DOWNWARD DIRECTION IS THE DIRECTION OF I ITSELF.

SINCE $\frac{dI}{dt}$ IS IN THE SAME DIRECTION AS I ,

$$I \text{ IS INCREASING}$$

TEST # 1
PHYSICS 122

NAME: _____

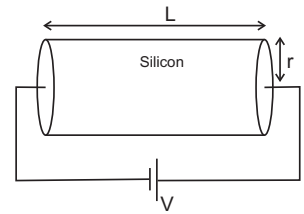
Do all five problems.

Show all work. Partial credit will be given for correct work.

Any constants that you need (such as mass or charge of an electron, or the value of ϵ_0) are available on the formula sheet, as are basic math formulas (areas, volumes, integrals, derivatives).

1 A piece of silicon (resistivity $2.3 \times 10^3 \, \Omega \, m$) is in the shape of a cylinder, of length $2 \, cm$ and radius $5 \, mm$. A $30 \, V$ battery is connected to the ends of the cylinder as shown.

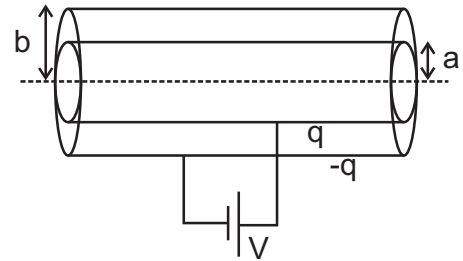
- (a) Which way is the current flowing? Which way are the electrons moving?
- (b) How many conduction electrons pass through the material per second?
- (c) How much heat is generated by the resistance in one minute?



2. A cylindrical capacitor consists of two thin, hollow metal cylinders, of radii a and b . A battery supplies a voltage difference V between the two cylinders, causing charges $\pm q$ to accumulate on them, as shown. Assume there is nothing but air in the gap of the capacitor, and that the cylinder is very long compared to the radii.

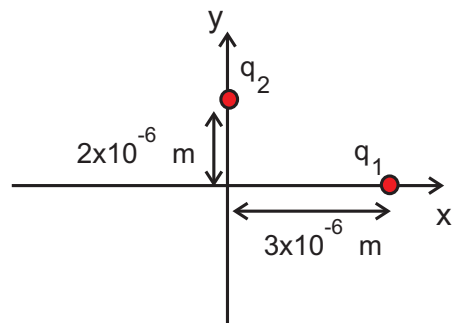
(a) *Derive* an expression for the electric field at distance r from the axis in the region between the cylinders ($a < r < b$). Write the answer in terms of the charge per length, $\lambda = \frac{q}{L}$. What direction does the field point?

(b) Using the result from part (a), *derive* an expression for the capacitance of the pair of cylinders. (Note that the result should be independent of λ , q , and V .)



3. Two **protons**, q_1 and q_2 , are located on the x - and y -axes, at the locations shown below. Find

- (a) Find the electric field (magnitude and direction) at the origin.
- (b) Find the voltage at the origin.
- (c) An *electron* is placed at the origin and released from rest. Find the magnitude and direction of its initial acceleration.

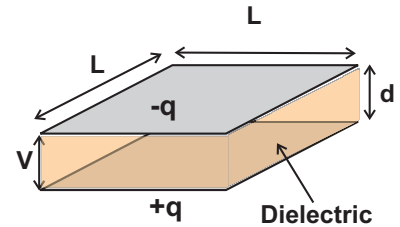


4. A parallel-plate capacitor consists of two square conducting plates. Each plate is $L = 0.03 \text{ m}$ long on each side, and they are separated by distance $d = 0.005 \text{ m}$. There is a dielectric between the plates, and a voltage difference of 30 V is applied across them.

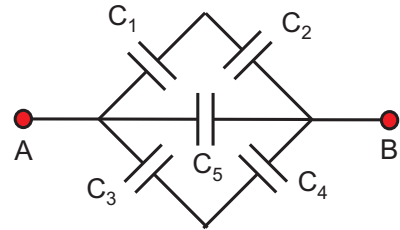
(a) What is the magnitude of the electric field in the gap? Which way does it point? material in the gap?

(b) If the energy density inside the capacitor is $3.186 \times 10^{-4} \frac{\text{J}}{\text{m}^2}$, what is the dielectric constant?

(c) Find the charge on the positive plate.



5. (a) Find the total capacitance of the following combination.
- (b) If a 24 V battery is connected between terminals A and B , what will be the voltage across each capacitor?
- (c) How much charge will collect on the plates of C_1 ?



$$C_1 = C_2 = C_3 = C_4 = C_5 = 4\ \mu\text{F}$$

TEST I SOLUTIONS

① $\rho = 2.3 \times 10^3 \Omega m$, $l = 0.02 m$, $r = 5 \times 10^{-3} m$, $V = 30 V$

a) CURRENT: COUNTER-CLOCKWISE
ELECTRONS: CLOCKWISE

b) $R = \frac{\rho l}{A} = \frac{(2.3 \times 10^3)(0.02)}{7.85 \times 10^{-5}} = 5.86 \times 10^5 \Omega$

$I = \frac{V}{R} = \frac{30}{5.86 \times 10^5} = 5.12 \times 10^{-5} A$

ELECTRONS PER SECOND:

$n = \left(\frac{I}{e} \right) \Delta t = \left(\frac{5.12 \times 10^{-5}}{1.6 \times 10^{-19}} \right) (1s) = 3.2 \times 10^{14}$

c) POWER: $P = I^2 R = (5.12 \times 10^{-5})^2 (5.86 \times 10^5) = 1.54 \times 10^{-3} W$

HEAT: $P \Delta t = (1.54 \times 10^{-3})(60) = 0.092 J$

② a) $\int \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \rightarrow E(2\pi r L) = \frac{q}{\epsilon_0}$

$E = \frac{q}{2\pi \epsilon_0 r L} = \left[\frac{\lambda}{2\pi \epsilon_0 r} \right] \text{ (OUTWARD)}$

b) VOLTAGE DIFFERENCE BETWEEN a & b:

$V = \left| - \int_a^b \vec{E} \cdot d\vec{r} \right| = \frac{\lambda}{2\pi \epsilon_0} \int_a^b \frac{dr}{r} = \frac{\lambda}{2\pi \epsilon_0} \ln \frac{b}{a}$

$q = CV \rightarrow C = \frac{q}{V} = \frac{\lambda L}{\frac{\lambda}{2\pi \epsilon_0} \ln \frac{b}{a}} = \left[\frac{2\pi \epsilon_0 L}{\ln \frac{b}{a}} \right]$

$$\textcircled{3} \text{ a) } E_y = -\frac{kq_2}{r_2^2} = -\frac{ke}{r_2^2} = -\frac{(9 \times 10^9)(1.6 \times 10^{-19})}{(2 \times 10^{-6})^2}$$

$$= -360 \frac{\text{V}}{\text{m}}$$

$$E_x = -\frac{kq_1}{r_1^2} = -\frac{(9 \times 10^9)(1.6 \times 10^{-19})}{(3 \times 10^{-6})^2} = -160 \frac{\text{V}}{\text{m}}$$

$$\boxed{\vec{E} = -160 \frac{\text{V}}{\text{m}} \hat{i} - 360 \frac{\text{V}}{\text{m}} \hat{j}} \rightarrow E = \sqrt{E_x^2 + E_y^2} = \boxed{393.9 \frac{\text{V}}{\text{m}}}$$

$$\theta = \tan^{-1} \frac{E_y}{E_x} = \boxed{246.0^\circ}$$

(4th QUADRANT)

$$\text{b) } V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$$

$$= (9 \times 10^9)(1.6 \times 10^{-19}) \left(\frac{1}{2 \times 10^{-6}} + \frac{1}{3 \times 10^{-6}} \right) = \boxed{1.2 \times 10^{-3} \text{ V}}$$

$$\text{c) } \vec{F} = q\vec{E} = -e\vec{E}$$

$$= -(1.6 \times 10^{-19}) (-160 \hat{i} - 360 \hat{j})$$

$$= 2.56 \times 10^{-17} \hat{i} + 5.76 \times 10^{-17} \hat{j} \quad (\text{IN N})$$

$$\vec{a} = \frac{\vec{F}}{m} = 1.53 \times 10^{14} \hat{i} + 3.45 \times 10^{14} \hat{j} \quad (\text{IN } \frac{\text{m}}{\text{s}^2})$$

MAG. & DIRECTION:

$$a = \sqrt{a_x^2 + a_y^2} = \boxed{6.92 \times 10^{13} \frac{\text{m}}{\text{s}^2}}$$

$$\theta = \tan^{-1} \frac{a_y}{a_x} = \boxed{66^\circ}$$

(OPPOSITE TO $\vec{E}$)

④ $L = .03 \text{ m} \rightarrow A = L^2 = 9 \times 10^{-4} \text{ m}^2$, $d = .005 \text{ m}$, $V = 30 \text{ V}$

a) $E = \frac{V}{d} = \frac{30}{.005} = \boxed{6000 \frac{\text{V}}{\text{m}}}$ POINTS UPWARD

b) $u = 3.186 \times 10^{-4} \frac{\text{J}}{\text{m}^3} = \frac{1}{2} \kappa \epsilon_0 E^2$

$\kappa = \frac{2u}{\epsilon_0 E^2} = \frac{2(3.186 \times 10^{-4})}{(8.85 \times 10^{-12})(6000)^2} = \boxed{2}$

c) $C = \frac{\kappa \epsilon_0 A}{d} = \frac{2(8.85 \times 10^{-12})(9 \times 10^{-4})}{.005}$
 $= 3.186 \times 10^{-12} \text{ F} (= 3.186 \text{ pF})$

$q = CV = (3.186 \times 10^{-12})(30) = \boxed{9.56 \times 10^{-11} \text{ C}}$

⑤ a) C_1 & C_2 IN SERIES $\rightarrow \frac{1}{C_A} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4} + \frac{1}{4} \Rightarrow C_A = 2 \mu\text{F}$

C_3 & C_4 IN SERIES $\rightarrow \frac{1}{C_B} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{4} + \frac{1}{4} \Rightarrow C_B = 2 \mu\text{F}$

C_A , C_B , & C_5 IN PARALLEL!

$C = C_A + C_B + C_5 = 2 + 2 + 4 = \boxed{8 \mu\text{F}}$

b) SINCE C_1 & C_2 HAVE THE SAME CAPACITANCE & THE SAME CHARGE (IN SERIES), THEY WILL HAVE THE SAME VOLTAGE; $V_1 = V_2$. BUT THEY ALSO MUST ADD UP TO $V = 24 \text{ V}$. SO:

$V_1 = V_2 = \frac{1}{2} V = \boxed{12 \text{ V}}$ $V_5 = \boxed{24 \text{ V}}$ ($V_3 = V_4 = 12 \text{ V}$)

c) THEN: $q_1 = C_1 V_1 = (4 \times 10^{-6} \text{ F})(12) = \boxed{4.8 \times 10^{-5} \text{ C}}$

$q_5 = C_5 V_5 = (4 \mu\text{F})(24 \text{ V}) = \boxed{96 \mu\text{C}}$ ($= 48 \mu\text{C}$)

TEST # 2
PHYSICS 122

NAME: _____

Do all six problems.

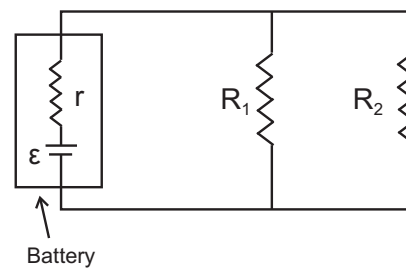
Show all work. Partial credit will be given for correct work.

Any constants that you need (mass or charge of an electron, value of ϵ_0 , etc.) are available on the formula sheet.

Basic math formula, including areas, volumes, integrals, derivatives are also on the formula sheet.

1. In the circuit below, the battery has an EMF of 20 V and internal resistance $r = 0.1\ \Omega$. The battery is connected in parallel to two resistors $R_1 = 4\ \Omega$ and $R_2 = 5\ \Omega$.

- (a) Find the size the current passing through the battery.
- (b) Find the voltage difference V between the two battery terminals.

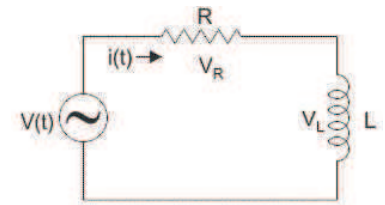


2. The power supply produces a time-varying voltage. At a particular moment, the power supply voltage is 30 V and the current in the circuit is 5 A , clockwise.

If the inductor has $L = 2\text{ H}$,

(a) Find the voltage across the inductor at the moment of interest.

(b) Find the rate at which the current is changing, $\frac{dI}{dt}$. Is I increasing or decreasing?

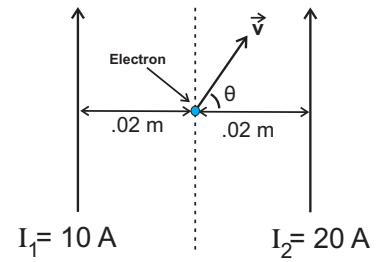


$R = 1\ \Omega$
 $L = 2\text{ H}$

At time t :
 $i(t) = 5\text{ A}$
 $V(t) = 30\text{ V}$

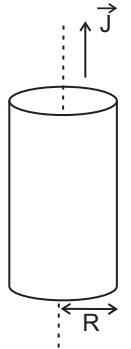
3. Two wires separated by a distance of $.04\text{ m}$ have currents $I_1 = 10\text{ A}$ and $I_2 = 20\text{ A}$. Both currents flow upward. At a particular moment, an electron is half-way between the wires, traveling with speed $v = 3 \times 10^4 \frac{\text{m}}{\text{s}}$ in the direction 60° from the horizontal.

- Find the **magnitude** and **direction** of the magnetic **field** at the electron's location.
- Find the **magnitude** of the magnetic **force** exerted by the wires on the electron.
- Find the **direction** of the magnetic **force** on the electron. Be specific: in particular make sure you specify the angle of the force from the horizontal.



4. A cable of radius $R = 0.015\text{ m}$ has a current passing through it, traveling upward. The current density is **not** uniform, but is a function of distance r from the axis: $J(r) = C \left(1 - \frac{r^2}{R^2}\right)$, where C is some known constant.

- (a) What is the total current, I ? (The answer will be a function of C .)
- (b) What is the magnitude of the magnetic field at the edge of the wire, $r = R$?
- (c) What is the direction of the magnetic field?

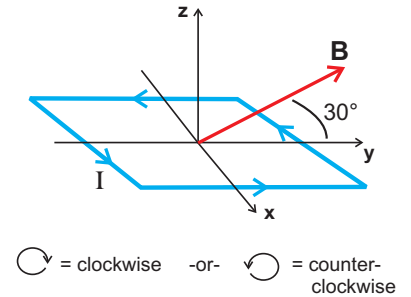


5. A square conducting loop (0.5 m on each side) lies horizontally. It carries a 20 A current in the direction shown, and is hinged so it can rotate around the x -axis. A uniform magnetic field $B = 4\text{ T}$ is pointing at 30° from the horizontal.

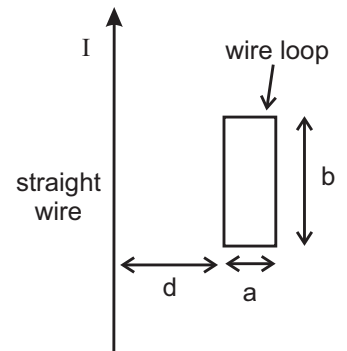
(a) If the loop, initially oriented as shown, is released from rest, what is the **magnitude and direction** of the initial torque? What direction will the loop rotate: clockwise or counterclockwise about the x -axis?

(b) If there is a small amount of friction, the loop will eventually come to rest. In what direction is the loop oriented when it finally comes to rest?

(c) What is the change in potential energy of the loop between its initial position and the position found in part (b)?



6. A long straight wire carries a current of I , flowing upward. At a distance of r from the wire, this produces a magnetic field of magnitude $B = \frac{I}{2\pi r}$ circulating around the wire
- (a) Suppose that a rectangular loop of width a and height b is placed a distance d from the wire. How large is the **magnetic flux** through the loop? (Note that **you need to do an integral** for this. It is an easy one.)
- (b) Using part (a), find the **mutual inductance** M between the wire and the loop.
- (c) If the current is **increasing** at a rate of $\frac{dI}{dt}$, how large is the induced EMF, $\mathcal{E}$ in the loop?
- (d) What is the **direction** of the induced current in the loop?



TEST 2 SOLUTIONS

① $\mathcal{E} = 20\text{V}$, $r = .1\ \Omega$, $R_1 = 4\ \Omega$, $R_2 = 5\ \Omega$

a) CAN SIMPLIFY! ADD R_1 & R_2 IN PARALLEL.

$$\frac{1}{R_{\parallel}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}$$

$$R_{\parallel} = \frac{20}{9}\ \Omega$$

so: $I = \frac{\mathcal{E}}{r + R_{\parallel}} = \frac{20}{.1 + \frac{20}{9}} = \frac{20}{.1 + 2.22} = \boxed{8.61\text{A}}$

(CAN ALSO USE KIRCHOFF RULES)

b) $V = \mathcal{E} - Ir = 20 - (8.61)(.1) = \boxed{19.14\text{V}}$

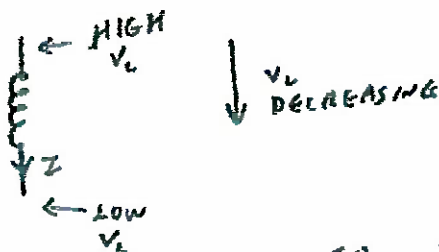
② a) KIRCHOFF LOOP RULE: $V - IR + V_L = 0$

$V_L = IR - V = (5)(1) - 30 = \boxed{-25\text{V}}$

↑ VOLTAGE ACROSS INDUCTOR (CAN ALSO CALL IT $\mathcal{E}$)
 ↓ DROPPING AS YOU GO CLOCKWISE AROUND LOOP.

b) $V_L = -L \frac{dI}{dt} \rightarrow \left| \frac{dI}{dt} \right| = \left| \frac{V_L}{L} \right| = \frac{25}{2} = \boxed{12.5 \frac{\text{A}}{\text{s}}}$

DIRECTION:



V_L IS PUSHING AGAINST THE CURRENT,

BY LENE, V_L OPPOSES CHANGE IN I .

SO THE CHANGE $\frac{dI}{dt}$ MUST BE IN THE SAME DIRECTION AS I .

THEREFORE, I IS INCREASING

③ $I_1 = 10 \text{ A}$, $I_2 = 20 \text{ A}$, $v = 3 \times 10^4 \frac{\text{m}}{\text{s}}$, $\theta = 60^\circ$, $r = .02 \text{ m}$

a) FIELDS FROM I_1 & I_2 AT ELECTRON POSITION:

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{(4\pi \times 10^{-7})(10)}{2\pi (.02)} = 10^{-4} \text{ T} \quad (\text{INTO PAGE})$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r} = \frac{(4\pi \times 10^{-7})(20)}{(2\pi)(.02)} = 2 \times 10^{-4} \text{ T} \quad (\text{OUT OF PAGE})$$

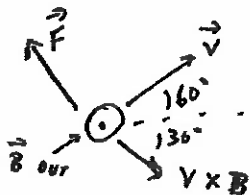
TOTAL FIELD:

$$B = |B_1 - B_2| = \boxed{10^{-4} \text{ T}} \quad (\text{OUT OF PAGE})$$

b) $\vec{F} = q \vec{v} \times \vec{B}$
 MAGNITUDE: $\vec{F} = |q v B \sin \phi| = e v B (\sin 90^\circ) = e v B$

$$= (1.6 \times 10^{-19})(3 \times 10^4)(10^{-4}) = \boxed{4.8 \times 10^{-19} \text{ N}}$$

c) DIRECTION: USE RIGHT-HAND RULE TO FIND $\vec{v} \times \vec{B}$, THEN FLIP SINCE ELECTRON CHARGE IS NEGATIVE:



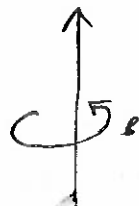
ANGLE FROM HORIZONTAL: $60^\circ + 90^\circ = \boxed{150^\circ}$
 (IN PLANE OF PAGE)

④ $R = 0.015 \text{ m}$, $J(r) = C = \text{CURRENT PER AREA}$

a) $I = \int J(r) dA = C \int dA = \boxed{C \pi R^2}$ ($= 7.07 \times 10^{-4} \text{ C}$, IN AMPS)

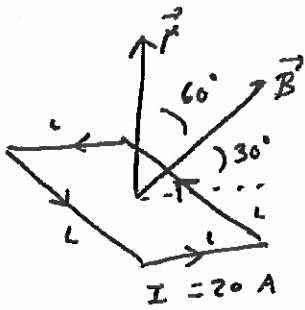
b) $\oint \vec{B} \cdot d\vec{s} = 2\pi R B$
 $\mu_0 I_{\text{enc}} = \mu_0 I = \mu_0 C \pi R^2$
 $\rightarrow 2\pi R B = \mu_0 C \pi R$
 $\boxed{B = \frac{1}{2} \mu_0 C R}$ ($= 9.4 \times 10^{-9} \text{ C}$, IN T)

c) DIRECTION OF $\vec{B}$:



COUNTER-CLOCKWISE AS VIEWED FROM ABOVE

5



$B = 4 \text{ T}$ AT 30° FROM HORIZONTAL.

$$\mu = IA = IL^2 = (20)(1.5)^2$$

$$= 5 \text{ A m}^2$$

($\vec{\mu}$ VERTICAL)

$$a) \tau = |\vec{\mu} \times \vec{B}| = \mu B \sin 60^\circ = (5)(4) \sin 60^\circ$$

$$= \boxed{17.32 \text{ N m}} \quad (\text{INTO PAGE})$$

ROTATION IS CLOCKWISE

b) LOOP ROTATES UNTIL $\vec{\mu}$ IS ALIGNED WITH $\vec{B}$,
(OR IN OTHER WORDS UNTIL LOOP IS $\perp$ TO $\vec{B}$).
SO LOOP ROTATES 60° CLOCKWISE.

$$c) \Delta U = U_{\text{FINAL}} - U_{\text{INITIAL}} = (-\mu B \cos 0^\circ) - (-\mu B \cos 60^\circ)$$

$$= -\mu B (\cos 0^\circ - \cos 60^\circ)$$

$$= -(5)(4) \left(1 - \frac{1}{2}\right)$$

$$= \boxed{-10 \text{ J}}$$

⑥ $B = \frac{\mu_0 I}{2\pi r} \rightarrow$ AT LOOPS LOCATION, $\vec{B}$ IS INTO THE PAGE.

$$\begin{aligned} \text{a) } \Phi &= \int \vec{B} \cdot d\vec{A} = \int_d^{d+a} B b dr = \int_d^{d+a} \frac{\mu_0 I b}{2\pi r} dr \\ &= \frac{\mu_0 I b}{2\pi} \int_d^{d+a} \frac{dr}{r} = \boxed{\frac{\mu_0 I b}{2\pi} \ln\left(\frac{d+a}{d}\right)} \end{aligned}$$

$$\text{b) } M = \frac{d\Phi}{dI} = \boxed{\frac{\mu_0 b}{2\pi} \ln\left(\frac{d+a}{d}\right)}$$

$$\text{c) } \frac{dI}{dt} > 0$$

$$|\mathcal{E}| = \left| -M \frac{dI}{dt} \right| = \boxed{\frac{\mu_0 b}{2\pi} \ln\left(\frac{d+a}{d}\right) \cdot \frac{dI}{dt}}$$

$$\text{d) } I \text{ IS } \boxed{\text{COUNTER-CLOCKWISE}}, \text{ BY LENZ' LAW}$$

REVIEW PROBLEMS

PHYSICS 122

Chapters 31-35

1. The average daytime intensity of sunlight at the surface of the earth is $1300 \frac{W}{m^2}$.
 - (a) Find the amplitude of the electric field in the sunlight.
 - (b) Find the rms value of the magnetic field.
 - (c) Find the maximum energy density of the sunlight.
 - (d) Find the pressure the sunlight exerts on a tar-covered driveway.
 - (e) Find the pressure the sunlight exerts on a white cement sidewalk.

2. An FM radio wave with frequency 10^8 Hz has a maximum electric field of $1.5 \times 10^{-2} \frac{V}{m}$.
 - (a) What is the maximum value of the magnetic field?
 - (b) What is the wavelength of the radio wave?
 - (c) The wave strikes an antenna dish with a surface area of 2 m^2 . What is the maximum power absorbed by the dish. (Assume that the wave is traveling perpendicular to the dish.)
 - (d) What is the average power absorbed by the antenna?
 - (e) What is the maximum energy density of the wave
 - (f) Electromagnetic waves can be viewed as collections of particles called photons. Each photon has energy hf , where $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$ is Planck's constant. What is the energy of each individual photon in this radio wave?
 - (g) Using the answers to parts d and f, find the average number of photons striking the antenna per second.

3. A light wave of frequency $f = 8 \times 10^{14} \text{ Hz}$ travels through the air until it hits a pane of glass. The glass has index of refraction of 1.6, and the light strikes it at 70° from the line perpendicular to the surface.
 - (a) What is the speed and wavelength of the light when it is in the air?
 - (b) What is the speed and wavelength when it is in the glass?
 - (c) At what angle from the perpendicular is the refracted beam?
 - (d) Is the reflected light polarized or not? $\leftarrow$ SKIP: DID NOT COVER
 - (e) The refracted beam continues on until it hits the other side of the glass. (The two sides of the pane are parallel.) Is the light totally internally reflected at the second surface, or is there a component transmitted back into the air on the other side of the glass?

4. A concave spherical mirror has a focal length of 10 cm . An object of height 4 cm is placed at a distance of 25 cm in front of the mirror.

- (a) Find the position of the image formed by the mirror.
- (b) Describe the image: Is it upright or inverted? Real or virtual? How tall is the image?
- (c) Repeat parts a and b, but with a convex mirror.

5. A converging lens has a focal length of 10 cm . An object of height 4 cm is placed 30 cm to the left of the lens.

- (a) Find the image formed by the lens.
- (b) Describe the image: Is it real or virtual? Upright or inverted? How tall?
- (c) Repeat parts a and b for a diverging lens.

6. A beam of unpolarized light of intensity I_0 passes through a polarizing sheet. It then strikes a second polarizer, with a polarization angle which is at 25° to the direction of the original sheet. Finally, it strikes a third one, with a polarizing direction at 25° to the second one. What is the intensity of the light (in multiples of I_0) after passing through each sheet?

7. Light emitted when an unknown element is heated passes through a grating spectrometer of 4000 slits per centimeter, forming two series of lines on a screen. The first line is violet and occurs at 9° from the center of the diffraction pattern. The second line (red) occurs at 17° . **(CAN SKIP THIS PROBLEM. WE DID NOT COVER IT.)**

- (a) Find the wavelengths of the two spectral lines.
- (b) Find the angular separation of the third order red and violet lines.

8. The luminosity (total emitted power) of the sun is $4 \times 10^{26}\text{ W}$. The average intensity of sunlight that reaches the earth is about $1300\frac{\text{W}}{\text{m}^2}$. In contrast, Sirius, the brightest star in the night sky looks about 10 billion times dimmer, with intensity of only $1.12 \times 10^{-7}\frac{\text{W}}{\text{m}^2}$ at earth. If Sirius is 5.44×10^5 astronomical units away (which just means it is 5.44×10^5 times farther from us than the sun), then how much more luminous is it than the sun? What is its total emitted power?

REVIEW PROBLEMS
CHS. 31-35

$$\textcircled{1} \text{ a) } \bar{S} = \frac{1}{2} \epsilon_0 c E_m^2 \rightarrow E_m = \sqrt{\frac{2\bar{S}}{\epsilon_0 c}} = \sqrt{\frac{2(1300)}{(8.85 \times 10^{-12})(3 \times 10^8)}} = \boxed{989.6 \text{ V/m}}$$

$$\text{b) } B_{rms} = \frac{1}{\sqrt{2}} B_m = \frac{1}{\sqrt{2}} \frac{E_m}{c} = \frac{989.6}{\sqrt{2} (3 \times 10^8)} = \boxed{2.33 \times 10^{-6} \text{ T}}$$

$$\text{c) } u_{max} = \epsilon_0 E_m^2 = (8.85 \times 10^{-12}) (989.6)^2 = \boxed{8.67 \times 10^{-6} \frac{\text{J}}{\text{m}^3}}$$

$$\text{d) } \text{BLACK} \rightarrow \text{ABSORBING} \rightarrow P_r = \frac{I}{c} = \frac{1300}{3 \times 10^8} = \boxed{4.3 \times 10^{-6} \text{ Pa}}$$

$$\text{e) } \text{WHITE} \rightarrow \text{REFLECTING} \rightarrow P_r = \frac{2I}{c} = \boxed{8.67 \times 10^{-6} \text{ Pa}}$$

$$\textcircled{2} \text{ a) } B_m = \frac{E_m}{c} = \frac{1.5 \times 10^{-2} \text{ V}}{3 \times 10^8 \frac{\text{m}}{\text{s}}} = \boxed{5 \times 10^{-11} \text{ T}}$$

$$\text{b) } f\lambda = c \rightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8 \frac{\text{m}}{\text{s}}}{10^8 \text{ Hz}} = \boxed{3 \text{ m}}$$

$$\text{c) } \text{POYNTING VECTOR: } S = \epsilon_0 c E^2$$

$$S_{max} = \epsilon_0 c E_m^2 = (8.85 \times 10^{-12}) (3 \times 10^8) (1.5 \times 10^{-2})^2 = \boxed{5.97 \times 10^{-7} \frac{\text{W}}{\text{m}^2}}$$

BUT THE MAGNITUDE OF S IS POWER PER AREA, SO:

$$P_{max} = S_{max} \cdot A = (5.97 \times 10^{-7} \frac{\text{W}}{\text{m}^2}) (2 \text{ m}^2) = \boxed{1.194 \times 10^{-6} \text{ W}}$$

$$\text{d) } \bar{P} = \frac{1}{2} P_{max} = \boxed{5.97 \times 10^{-7} \text{ W}}$$

$$\text{e) } u = \epsilon_0 E^2 \rightarrow u_{max} = \epsilon_0 E_m^2 = (8.85 \times 10^{-12}) (1.5 \times 10^{-2})^2 = \boxed{1.99 \times 10^{-15} \frac{\text{J}}{\text{m}^3}}$$

$$\text{f) } E = hf = (6.63 \times 10^{-34} \text{ J}\cdot\text{s}) (10^8 \text{ Hz}) = \boxed{6.63 \times 10^{-26} \text{ J}}$$

$$\text{g) } (\# \text{ PHOTONS PER SECOND}) = \frac{\text{ENERGY PER SECOND}}{\text{ENERGY PER PHOTON}} \\ = \frac{5.97 \times 10^{-7} \text{ W}}{6.63 \times 10^{-26} \text{ J}} = \boxed{9.0 \times 10^{18} \text{ PHOTONS/SEC}}$$

③ a) $v_{\text{AIR}} = \frac{c}{n_{\text{AIR}}} = \frac{c}{1} = c = \boxed{3 \times 10^8 \frac{\text{m}}{\text{s}}}$

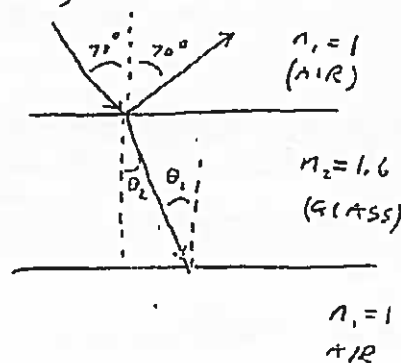
$v_{\text{AIR}} = f \lambda_{\text{AIR}} \Rightarrow \lambda_{\text{AIR}} = \frac{v_{\text{AIR}}}{f} = \frac{3 \times 10^8}{8 \times 10^{14}} = \boxed{3.75 \times 10^{-7} \text{ m}}$

b) $v = \frac{c}{n} = \frac{3 \times 10^8}{1.6} = \boxed{1.875 \times 10^8 \frac{\text{m}}{\text{s}}}$ $\lambda = \frac{v}{f} = \frac{1.875 \times 10^8}{8 \times 10^{14}} = \boxed{2.34 \times 10^{-7} \text{ m}}$

(NOTE THAT f IS THE SAME IN BOTH MATERIALS)

c) $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$\theta_2 = \sin^{-1} \left[\frac{n_1}{n_2} \sin \theta_1 \right] = \sin^{-1} \left[\frac{1}{1.6} (\sin 70^\circ) \right]$
 $= \boxed{36.0^\circ}$



d) BREWSTER ANGLE:
 $\theta_p = \tan^{-1} \frac{n_2}{n_1} = \tan^{-1} \left(\frac{1.6}{1} \right) = 58.0^\circ$

SINCE $\theta_1 \neq \theta_p$, THE REFLECTED LIGHT IS NOT POLARIZED.

e) CRITICAL ANGLE: $\theta_c = \sin^{-1} \frac{n_{\text{AIR}}}{n_{\text{GLASS}}} = \sin^{-1} \left(\frac{1}{1.6} \right) = 38.68^\circ$

SINCE $\theta_2 < \theta_c$ - THERE IS NO TOTAL INTERNAL REFLECTION. (PART OF THE BEAM CONTINUES OUT OF THE GLASS)

④ a) $f = 10 \text{ cm}$, $h_o = 4 \text{ cm}$, $d_o = 25 \text{ cm}$

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}} = \frac{1}{\frac{1}{10} - \frac{1}{25}} = \boxed{16.67 \text{ cm}}$
 IN FRONT OF MIRROR, SINCE $d_o > 0$.

b) $m = -\frac{d_i}{d_o} = -\frac{16.67}{25} = -.67$

IMAGE IS REAL (SINCE $d_i > 0$) & INVERTED (SINCE $m < 0$),

HEIGHT: $h_i = m h_o = (-.67)(4 \text{ cm}) = \boxed{-2.68 \text{ cm}}$ ← MINUS SIGN MEANS IMAGE IS INVERTED

c) NOW, $f = -10 \text{ cm}$

$d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}} = \frac{1}{-\frac{1}{10} - \frac{1}{25}} = \boxed{-7.14 \text{ cm}}$ $m = -\frac{d_i}{d_o} = -\left(\frac{-7.14}{25}\right) = .286$
 IMAGE IS BEHIND MIRROR

$m > 0 \Rightarrow$ UPRIGHT

$d_i < 0 \Rightarrow$ VIRTUAL

$h_i = m h_o = (.286)(4 \text{ cm}) = \boxed{1.144 \text{ cm}}$

5) a) $f = +10 \text{ cm}$, $h_o = 4 \text{ cm}$, $d_o = 30 \text{ cm}$

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}} = \frac{1}{\frac{1}{10} - \frac{1}{30}} = +15 \text{ cm}$

THE POSITIVE SIGN MEANS ON THE SIDE OF THE OUTGOING LIGHT, (TO THE RIGHT OF THE LENS)

b) $m = -\frac{d_i}{d_o} = -\frac{15}{30} = -\frac{1}{2}$

$m < 0 \Rightarrow \text{INVERTED}$

$d_i > 0 \Rightarrow \text{REAL}$

$h_i = m h_o = (-\frac{1}{2})(4 \text{ cm}) = -2 \text{ cm}$

UPSIDE-DOWN

c) NOW, $f = -10 \text{ cm}$.

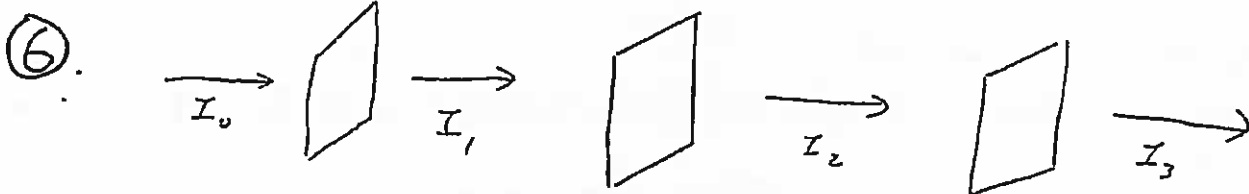
$d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}} = \frac{1}{\frac{1}{-10} - \frac{1}{30}} = -7.5 \text{ cm}$

← TO LEFT (ON SIDE OPPOSITE OUTGOING LIGHT) BECAUSE OF MINUS SIGN.

$d_i < 0 \Rightarrow \text{VIRTUAL}$

$m = -\frac{d_i}{d_o} = -\frac{(-7.5)}{30} = .25 > 0 \Rightarrow \text{UPRIGHT}$

$h_i = m h_o = (.25)(4) = 1 \text{ cm}$



SINCE THE ORIGINAL BEAM IS UNPOLARIZED, ONE OF THE 2 POSSIBLE POLARIZATION COMPONENTS IS LOST GOING THROUGH THE FIRST POLARIZER:

$I_1 = \frac{1}{2} I_0$

2nd POLARIZER: USE LAW OF MALUS, WITH $\theta = 25^\circ$.

$I_2 = I_1 \cos^2 25^\circ = (\frac{1}{2} I_0) \cos^2 25^\circ = .41 I_0$

3rd POLARIZER: DO THE SAME AGAIN.

$I_3 = I_2 \cos^2 25^\circ = (.41 I_0) \cos^2 25^\circ = .337 I_0$

⑦ DISTANCE BETWEEN LINES OF GRATING:

$$d = \frac{1}{4000 \text{ cm}^{-1}} = 2.5 \times 10^{-4} \text{ cm} = 2.5 \times 10^{-6} \text{ m}$$

a) ORDER: $m=1 \rightarrow$ ANGLES: $\begin{cases} \theta_1 = 9^\circ & \text{VIOLET} \\ \theta_2 = 17^\circ & \text{RED} \end{cases}$

$$\sin \theta = \frac{m\lambda}{d}, \text{ so:}$$

$$\text{VIOLET: } \lambda_1 = \frac{d}{m} \sin \theta_1 = \frac{2.5 \times 10^{-6} \text{ m}}{1} \sin 9^\circ \\ = 3.9 \times 10^{-7} \text{ m} = \boxed{390 \text{ nm}}$$

$$\text{RED: } \lambda_2 = \frac{d}{m} \sin \theta_2 = \frac{2.5 \times 10^{-6} \text{ m}}{1} \sin 17^\circ \\ = 7.3 \times 10^{-7} \text{ m} = \boxed{730 \text{ nm}}$$

b) $m=3$:

$$\Delta \theta = \theta_2 - \theta_1 = \sin^{-1}\left(\frac{m\lambda_2}{d}\right) - \sin^{-1}\left(\frac{m\lambda_1}{d}\right)$$

$$= \sin^{-1}\left(\frac{3(7.3 \times 10^{-7})}{2.5 \times 10^{-6}}\right) - \sin^{-1}\left(\frac{3(3.9 \times 10^{-7})}{2.5 \times 10^{-6}}\right) = 61.1^\circ - 27.9^\circ \\ = \boxed{33.2^\circ}$$

⑧ SUN: $P_{\text{sun}} = 4 \times 10^{26} \text{ W}$, $I_{\text{sun}} = 1300 \frac{\text{W}}{\text{m}^2}$ (AT EARTH)

SIRIUS: ~~P_{sun}~~ $r_{\text{SIRIUS}} = 5.44 \times 10^5 r_{\text{sun}}$, $I_{\text{SIRIUS}} = 1.12 \times 10^{-7} \frac{\text{W}}{\text{m}^2}$

$$P_{\text{SIRIUS}} = 4\pi r_{\text{SIRIUS}}^2 I_{\text{SIRIUS}} = (5.44 \times 10^5)^2 (4\pi r_{\text{sun}}^2) I_{\text{SIRIUS}} \\ = (5.44 \times 10^5)^2 \left(\frac{P_{\text{sun}}}{I_{\text{sun}}}\right) I_{\text{SIRIUS}} \quad \leftarrow \left(\text{USING } I_{\text{sun}} = \frac{P_{\text{sun}}}{4\pi r^2}\right) \\ = (5.44 \times 10^5)^2 \left(\frac{4 \times 10^{26}}{1300}\right) (1.12 \times 10^{-7}) \\ = \boxed{1.02 \times 10^{28} \text{ W}}$$

$$\boxed{\frac{P_{\text{SIRIUS}}}{P_{\text{sun}}} = 25.4}$$

$\rightarrow$ EVEN THOUGH SIRIUS LOOKS 10 BILLION TIMES DIMMER THAN SUN, IT IS ACTUALLY 25 TIMES BRIGHTER. IT JUST LOOKS LESS BRIGHT DUE TO ITS GREATER DISTANCE.

PRACTICE

**FINAL EXAM
PHYSICS 122**

NAME: _____

Do any six of the seven problems.

Show all work. Partial credit will be given for correct work.

1. A star emits a total optical power of $5 \times 10^{26} W$. A three-headed astronomer named Zrg resides on the planet Xltn, which orbits this star at a distance of $r = 2 \times 10^{12} m$. Zrg is making measurements of the incoming starlight.

(a) What is the intensity of the starlight that arrives at the surface of the planet?

(b) What is the *maximum* electric field E_m and the *rms* magnetic field B_{rms} that Zrg measures?

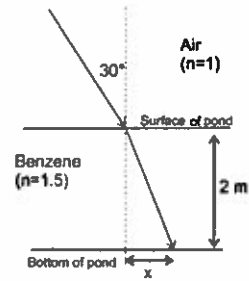
(c) What is the *average* force that the starlight exerts on the white roof of the Xltnian Astronomy Institute? (The roof has area $A = 3000 m^2$.)

2. Next to the Xltnian Astronomy Institute is a small benzene pond, where Zrg's pet slime-creature Fred lives. Fred knows that it is dinner time when the starlight hits the surface of the lake at 30° from the vertical. (Benzene has index of refraction 1.50. Assume the planet's atmosphere has index 1.0.)

(a) At dinner time, how far horizontally (x) does the light travel inside the benzene before hitting the bottom of the pond? (The pond is 2 m deep.)

(b) How fast does the light travel through the benzene?

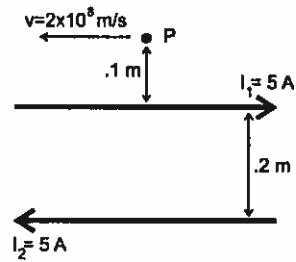
(c) If the light has wavelength $\lambda_1 = 500 \text{ nm}$ in air, what is its *wavelength* inside the pond? (Note that the *frequency* of the light does not change. This is because frequency and energy are proportional, and energy is conserved.)



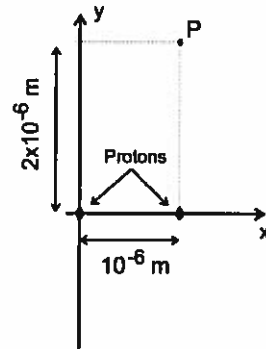
3. Two long straight wires separated by $.2\text{ m}$ carry equal currents (5 A) in opposite directions, as shown.

(a) Find the magnetic force (magnitude and direction) that the wires exert on an electron moving toward the left at $2 \times 10^8 \frac{\text{m}}{\text{s}}$ at point P .

(b) Find the magnetic force (magnitude and direction) that the top wire exerts on a 2 m long segment of the bottom wire.



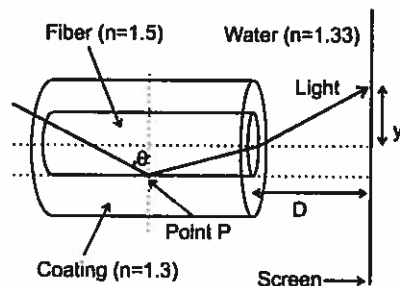
4. Two protons (charge $q = +e = 1.6 \times 10^{-19} \text{ C}$) are at the positions shown.
- (a) What is the electric potential (voltage) at point P ?
- (b) Find the magnitude and direction of the force that would be exerted on an electron ($q = -e$) if it was placed at point P .



5. Light inside an optical fiber strikes the outer edge of the fiber at an angle of $\theta = 40^\circ$ (point P in the figure). The indices of refraction in the fiber and in the surrounding material are given in the figure.

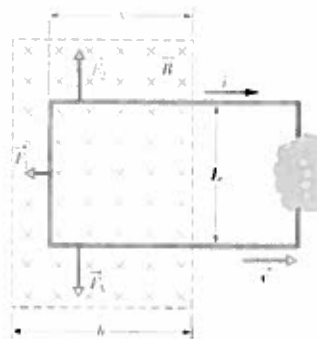
(a) Does total internal reflection occur when the light reflects off the outer coating (at point P)?

(b) At the end of the fiber, the light exits the fiber and enters water, hitting a screen $D = .08 \text{ m}$ away. At what vertical distance y from the fiber axis does the light hit the screen? (Assume the light is crossing the fiber axis when it reaches the fiber's end.)

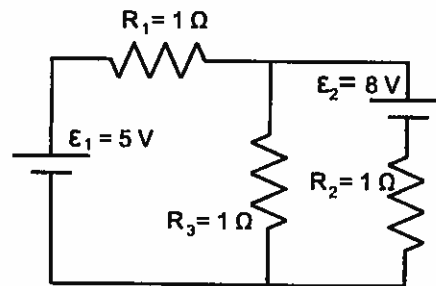


6. A rectangular wire loop of height $L = .4 \text{ m}$ in the vertical direction is pulled to the right at constant speed $v = 1.5 \frac{\text{m}}{\text{s}}$. The left-hand part of the loop is inside a region of width $b = .3 \text{ m}$ that contains a uniform magnetic field of magnitude $B = 20 \text{ T}$ into the page. The loop has resistance $R = 6 \Omega$.

- Find the *size* and *direction* of the induced current in the loop.
- Find the magnitude of the force F_1 that the field exerts on the left-hand segment of the loop.



7. Set up the equations that need to be solved for the all of the unknown currents in circuit below. (Don't try to solve the equations.) Clearly label the unknown currents on the picture, including the directions that you assumed when writing down the equations.



FINAL EXAM SOLUTIONS

① $P = 5 \times 10^{26} \text{ W}$, $r = 2 \times 10^{12} \text{ m}$, $A = 3000 \text{ m}^2$

a) $I = \frac{P}{4\pi r^2} = \frac{5 \times 10^{26}}{4\pi (2 \times 10^{12})^2} = \boxed{9.95 \frac{\text{W}}{\text{m}^2}}$

b) $\bar{u} = \frac{I}{c} = 3.32 \times 10^{-8} \text{ J/m}^3$

$\bar{u} = \frac{1}{2} \epsilon_0 E_m^2 \rightarrow E_m = \sqrt{\frac{2\bar{u}}{\epsilon_0}} = \sqrt{\frac{2(3.32 \times 10^{-8})}{8.85 \times 10^{-12}}}$

$= \boxed{86.6 \frac{\text{V}}{\text{m}}}$

$B_{\text{rms}} = \frac{B_0}{\sqrt{2}} = \frac{E_m}{\sqrt{2} c}$

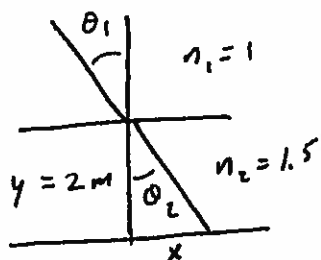
c) ROOF IS WHITE (REFLECTING):

$F = P_r A = 2\bar{u} A = 2(3.32 \times 10^{-8})(3000)$

$= \boxed{1.99 \times 10^{-4} \text{ N}}$

$= \boxed{2.04 \times 10^{-7} \text{ T}}$

②



a) $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$\theta_2 = \sin^{-1} \left(\frac{n_1}{n_2} \sin \theta_1 \right)$

$= \sin^{-1} \left(\frac{1}{1.5} \sin 30^\circ \right) = \boxed{19.5^\circ}$

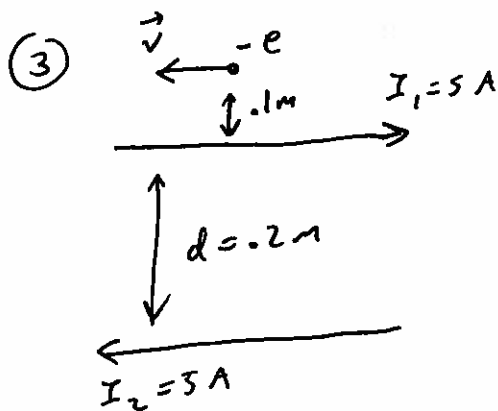
$\tan \theta_2 = \frac{x}{y} \rightarrow x = y \tan \theta_2 = 2 \tan 19.5^\circ$

$= \boxed{0.708 \text{ m}}$

b) $v = \frac{c}{n} = \frac{3 \times 10^8}{1.5} = \boxed{2 \times 10^8 \frac{\text{m}}{\text{s}}}$

c) $f = \frac{c}{\lambda_1} = \frac{3 \times 10^8}{5.0 \times 10^{-7}} = \boxed{6.0 \times 10^{14} \text{ Hz}}$

$\lambda_2 = \frac{v}{f} = \frac{2 \times 10^8}{6.0 \times 10^{14}} = 3.33 \times 10^{-7} \text{ m}$
 $= \boxed{333 \text{ nm}}$



$$v = 2 \times 10^8 \frac{m}{s}$$

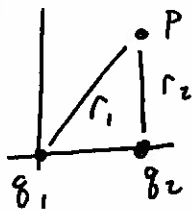
$$\begin{aligned} a) B &= B_1 - B_2 = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} \\ &= \frac{\mu_0}{2\pi} \left(\frac{I_1}{r_1} - \frac{I_2}{r_2} \right) \\ &= \frac{4\pi \times 10^{-7}}{2\pi} \left(\frac{5}{.1} - \frac{5}{.3} \right) \\ &= \boxed{6.67 \times 10^{-6} T} \quad (\text{OUT}) \end{aligned}$$

$$\begin{aligned} F &= |q \vec{v} \times \vec{B}| = (1.6 \times 10^{-19}) (2 \times 10^8) (6.67 \times 10^{-6}) \\ &= \boxed{2.13 \times 10^{-16} N} \quad \underline{\text{DOWNWARD}} \end{aligned}$$

$$b) B_1 = \frac{\mu_0 I_1}{2\pi d} = 5 \times 10^{-6} T$$

$$F = I_2 l_2 B_1 = (5)(2)(5 \times 10^{-6}) = \boxed{5 \times 10^{-5} N} \quad (\text{DOWN})$$

(4) a)



$$r_1 = \sqrt{(2 \times 10^{-6})^2 + (10^{-6})^2} = 2.24 \times 10^{-6} m$$

$$r_2 = 2 \times 10^{-6} m$$

$$\begin{aligned} V &= \frac{kq}{r_1} + \frac{kq}{r_2} = (9 \times 10^9)(1.6 \times 10^{-19}) \\ &\quad \times \left(\frac{1}{2.24 \times 10^{-6}} + \frac{1}{2 \times 10^{-6}} \right) \\ &= \boxed{1.36 \times 10^{-3} V} \end{aligned}$$

PART 6
NEXT PAGE

b) MAGNITUDES OF SEPARATE FIELDS:

$$E_1 = \frac{kq}{r_1^2} = 286.99 \frac{V}{m} \quad \text{STRAIGHT UP } (\theta_1 = +90^\circ)$$

$$E_2 = \frac{kq}{r_2^2} = 360 \frac{V}{m} \quad \text{AT } \theta_2 = \tan^{-1} \frac{2 \times 10^{-6}}{10^{-6}} = 63.4^\circ$$

COMPONENTS OF TOTAL FIELD:

$$E_x = E_{1x} + E_{2x} = E_1 \cos \theta_1 + E_2 \cos \theta_2$$

$$= 287 \cos 90 + 360 \cos 63.4^\circ = 448.2 \frac{V}{m}$$

$$E_y = E_{1y} + E_{2y} = E_1 \sin \theta_1 + E_2 \sin \theta_2$$

$$= 287 \sin 90 + 360 \sin 63.4 = 608.9 \frac{V}{m}$$

MAGNITUDE & DIRECTION:

$$E = \sqrt{E_x^2 + E_y^2} = \boxed{756 \frac{V}{m}} \quad \theta = \tan^{-1} \frac{E_y}{E_x} = \boxed{53.6^\circ}$$

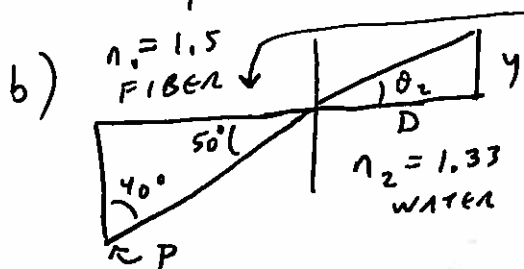
SO FORCE IS:

$$\vec{F} = -e\vec{E} \rightarrow F = (1.6 \times 10^{-19})(756) = \boxed{1.21 \times 10^{-16} N}$$

$$\text{AT } \theta = 53.6^\circ + 180^\circ = \boxed{233.6^\circ}$$

⑤ a) $\theta_c = \sin^{-1} \frac{n_2}{n_1} = \sin^{-1} \frac{1.3}{1.5} = 60.07^\circ$

$\theta_1 = 40^\circ$. SINCE $\theta_1 < \theta_c \Rightarrow \boxed{\text{NO TOT. INT. REFL.}}$



NOW $\theta_1 = 50^\circ$ AT INTERFACE WITH WATER. $n_2 \sin \theta_2 = n_1 \sin \theta_1$

SO:

$$\theta_2 = \sin^{-1} \left(\frac{n_1}{n_2} \sin \theta_1 \right) = \sin^{-1} \left(\frac{1.5}{1.33} \sin 50 \right)$$

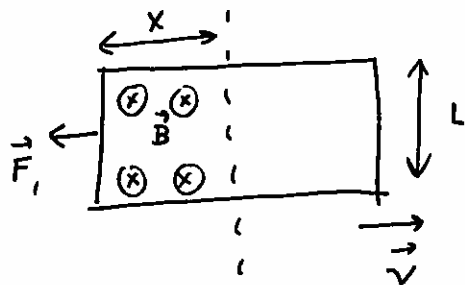
$$= 59.76^\circ$$

SO FROM THE RIGHT TRIANGLE IN THE WATER:

$$\tan \theta_2 = \frac{y}{D} \rightarrow y = D \tan \theta_2 = (0.08) \tan 59.76^\circ$$

$$= \boxed{0.137 m}$$

⑥



$$L = .4 \text{ m}, \quad v = 1.5 \frac{\text{m}}{\text{s}}, \quad R = 6 \Omega$$

$$B = 20 \text{ T}, \quad N = 1$$

$$a) |\mathcal{E}| = \left| N \cdot \frac{d\Phi_B}{dt} \right| = \frac{d}{dt}(BA) = B \frac{dA}{dt} = BL \frac{dx}{dt}$$

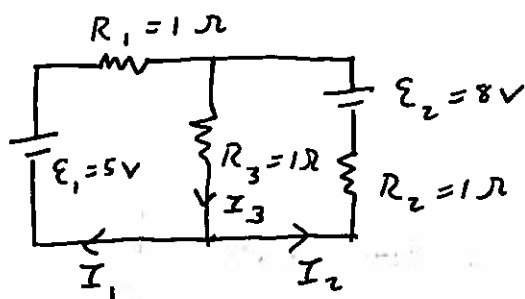
$$= BLv = (20)(.4)(1.5) = \boxed{12.0 \text{ V}}$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{12}{6} = \boxed{2 \text{ A}} \quad \boxed{\text{CLOCKWISE}}$$

(BY LENZ)

$$b) F_1 = LBI = (.4)(20)(2) = \boxed{16 \text{ N}}$$

⑦



WITH THE CURRENTS
DEFINED AS SHOWN:

$$\boxed{I_1 + I_2 = I_3} \quad (\text{JUNCTION RULE})$$

AND

$$\boxed{\begin{aligned} \mathcal{E}_1 - I_1 R_1 - I_3 R_3 &= 0 \\ \mathcal{E}_2 - I_3 R_3 - I_2 R_2 &= 0 \end{aligned}} \quad (\text{LOOP RULE})$$

(IF YOU DEFINED THE CURRENTS DIFFERENTLY,
THE EQUATIONS MAY LOOK SLIGHTLY DIFFERENT)

IF YOU PLUG IN THE VALUES, THE LAST 2
EQUATIONS BECOME:

$$\boxed{\begin{aligned} 5 - I_1 - I_3 &= 0 \\ 8 - I_3 - I_2 &= 0 \end{aligned}}$$

GOING AHEAD & SOLVING THIS SET OF EQUATIONS
ISN'T VERY HARD.